HERMANN KIRCHNER, director of a home for mentally handicapped children in Hepsisau, Germany, draws here upon an experience of several decades. Thus he approaches Dynamic Drawing, or Form Drawing as it is also called, from a therapeutic viewpoint. His work supplements the Form Drawing as used in all Waldorf Schools for mentally healthy children. (See *Form Drawing* by Niederhauser and Frohlich, published 1974 by the Rudolf Steiner School in New York).

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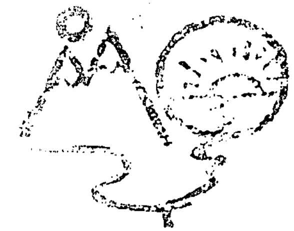

DYNAMIC DRAWING

Its Therapeutic Aspect

The practice of an art—as also the experience of it—presupposes a gathering of one's inner forces, a harmony that comes about through inner concentration. The arts with their variety of media give a teacher broad opportunities for their use in curative work. The constant creative effort in continual repetition which art requires, makes the harmony which has come about spread over into everyday life. Since various senses are involved in practicing and also in contemplating the different arts the teacher can adapt their therapeutic value to different illnesses. In this essay we will speak of dynamic drawing. Neither color nor a portrayal of nature will be considered. We will be dealing altogether with the drawing of lines which come about through movement, which therefore are its visible counterpart.

In many cases a retarded child has difficulty in entering actively into earth life with his whole sense organism. So-called "feeble-mindedness" points to the inability of the child's soul and spirit being to penetrate his physical body, thus preventing his having a complete relationship to the world around him through his sense organism. Awkward heaviness of the physical organism, dullness of the senses in relating himself to the surrounding world are the result. In the fifth lecture of the course on *CURATIVE EDUCATION* Rudolf Steiner says: "If you are dealing with a feeble-minded child you must bring his metabolic limb system into movement. This will stimulate his spiritual entity." We must then ask ourselves the question: Where in the realm of art do we work directly with the sense of movement? In Rudolf Steiner's analysis of the senses he establishes the existence of

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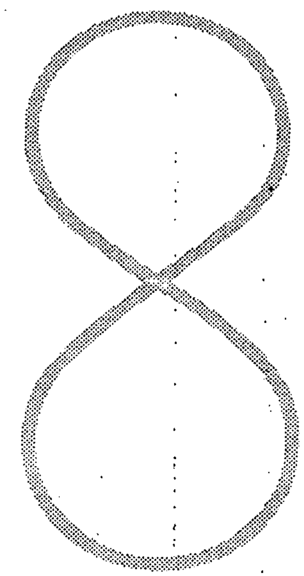


Translated by Margaret Frohlich

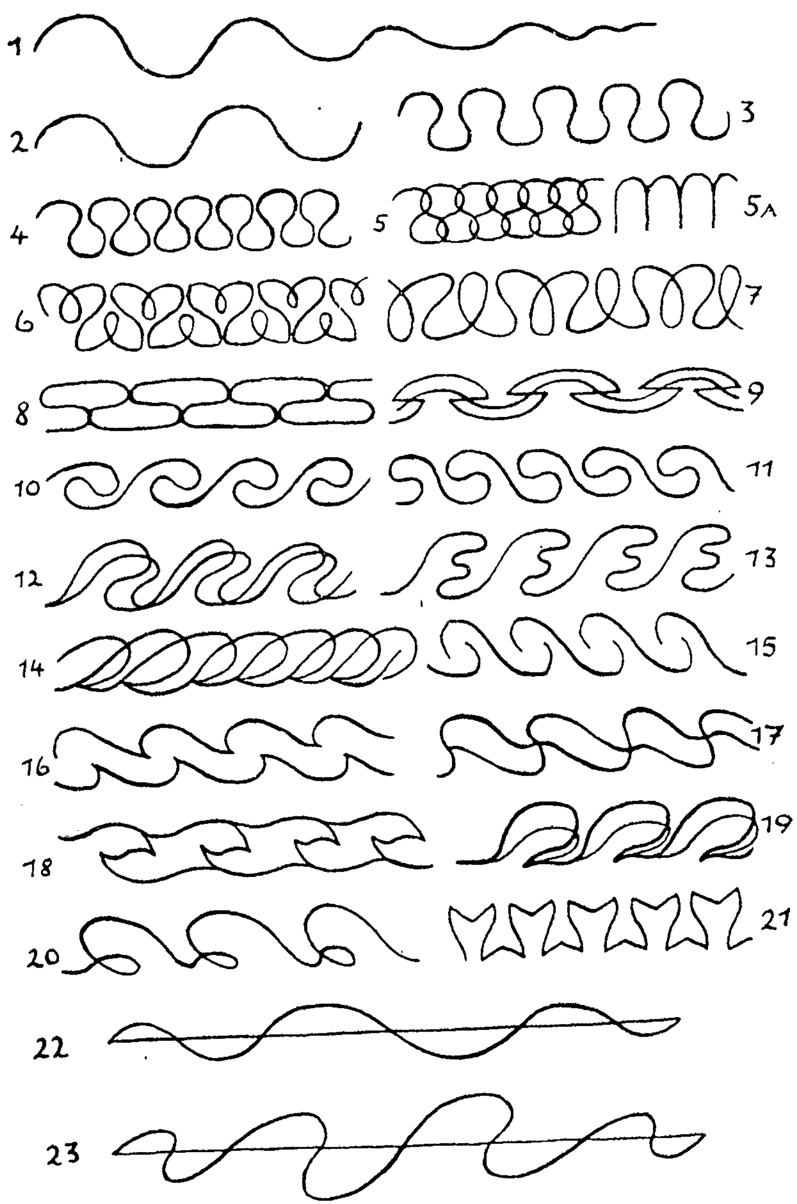
several senses, clearly distinguishable, in addition to those generally accepted. The sense of movement is one of them; this sense has the task of perceiving all one's own movement and all movement that takes place outside of one. This includes the experiencing of all forms, for they are all movements that have come to rest. While our sense of vision registers the color of a round surface our sense of movement feels its way around the periphery. If the periphery is an exact geometrical circle we have an harmonious experience that leads us unconsciously to cosmic laws. But if the circle is imperfect at a certain point, everyone will at once be aware of the flaw. Our sense of balance has been involved, and our impulse to correct the inexact spot shows that we are slightly disturbed in the well-being of our life sense. Similarly all existing forms have an effect on our sense of balance through our sense of movement. The reaction of our life sense shows to what degree a form, whether derived from nature or from art, is harmonious.

It is essential that the teacher who will be bringing the world of form to the child through his daily art work should be aware of its laws. Viewed from the earth both sun and moon appear to our naked eye as perfectly round plane surfaces. As can be observed on the edge of a shadow, their light appears to fall in a perfectly straight line. So the circle and the straight line are form elements coming to us from the cosmos. The rigidity of the straight line is weakened as the light nears the earth. Each degree of density it penetrates causes deviation from the original direction and reduces the rigidity. The ocean surfaces that make a partial sphere around our earth demonstrate the working of cosmic laws of form. In the linear surfaces of crystals one can discern formative activity working in from outer space. And in man the rounded surface of the eye reminds us that it was once formed by light streaming in from the cosmos. A similar origin must needs lie at the root of all cosmic-geometric forms. However, in art that is expressing a purely human element this experience finally ends. In so far as the artist does not use geometrical forms he enters a realm of freedom.

An artist in creating his forms can take his observation of nature as his starting point; then his work is more or less imitative. The more he converts the forms he finds into something new, out of his own experience, the more creative his work will be, and consequently, the more artistic. The contemporary artist has frequently gone the opposite way. He starts out from his inner state of soul and looks creatively for a corresponding symbol. In this case, if his work is carried out with sufficient wakefulness of will he is very often able to give interpretations of nature which do not derive from his remembered sense perceptions. Rather they manifest something essential in the forms themselves, making visible gestures which previously have been experienced in the surrounding world but which now stem from the depth of our supersensible being.

The following question arises for the teacher: what forms am I to choose and how do I present them to a child? Rudolf Steiner indicates that if we exclude forms from nature, the child as he experiences the movements of his arms and hands arrives at forms which stem from the laws of human movement. Linear drawing that has this idea as its starting point remains creative and alive and finds forms that are related to the human being. It will not create forms for form's sake. The line that is just the contour of a visible object is inartistic, untrue and does not exist in nature. Here we are dealing with the line as the graphic trace of a movement which is becoming visible on paper as it is carried out with a crayon. The child who exerts himself in this activity arrives rather quickly at the limit of his resources. Constant repetition of form elements that derive from his instinct or, as happens with a child who needs curative help, chaotic movements that derive from his physical condition, warn the teacher to look for a way to lead him beyond his mere impulsive drawing to a conscious shaping of forms. Whoever has developed the ability through eurythmy to read and interpret movements can learn a great deal about someone's personality by studying the character of his movements. The gestures make the illness visible; they are valuable in diagnosis. Therapy begins when the child wrestles with the movement of his crayon to achieve something of which he is incapable without this effort. The therapeutic value lies in this persistent struggle against his inner chaos.

What actually happens can best be observed in that group of children who suffer from moving too much, whose movements indicate that their ego is not in control. Their movements do not belong in the realm of play or in any sensible activity. There are many reasons for the illness of these hyperactive children. In addition to those whose constitution can be traced to their individual destiny there are all those children who have been injured by modern



technology. These are borderline cases in normal schools where they disturb the classes with their constant restlessness. That is why they are temporarily in need of a curative home. At the beginning of this century the motor was not yet part of our everyday surroundings, but today there is hardly a remote corner where it is not visible or audible. Its noise has no rhythm; it remains a foreign body in the human sphere. The noise cannot even be kept away from the baby carriage. Later on the child's gaze follows its source with fascination. With the little ones the result of constantly seeing and hearing a machine is that while they run about they themselves make motor noises in order to have them when no actual machine happens to be audible. The urge lies in them constantly to experience vibration. In this state the children come to school and with no inner tranquility they are unable to listen to the human voice of their teacher. Nor do they listen to their parents at home. In this unruly behavior the teacher can recognize that their movements are not true gestures, that is, they do not come from the spiritual core of these young beings. And so the teacher has the task of transforming all their soul-less mechanical movements into what is genuinely human. When these children are asked to draw a wave line it will look approximately like Figure 1: the first few waves still reflect the teacher's instruction; after that it rushes on in their own uncontrolled way. Within minutes the page is filled and the child has only repeated on paper the type of movement that he has been making all day. It is now important that the teacher should draw a wave line for him with as much precision as possible. As he draws he should speak of the controlled movements of his hand, how it carefully makes the transition from the upward curve to the downward curve. (Figure 2.) Once he succeeds in getting the child's attention to this process he can then very soon get him to exert some effort of his own. At this point there will be silence. It is only possible to draw a wave line that is geometrically exact, consisting of semicircles that merge into one another, if one consciously controls one's every movement. It will take quite half an hour or longer for a child to fill a page in his book that formerly he filled in a few minutes. Now the child's ego is taking charge. Frequent practice leads to perseverance.

As soon as the child has developed some skill the teacher gives him additional forms to draw. The child's uncontrolled onrush helps to find them. The teacher's desire to control this onrushing impulse will bring him to forms that will develop out of one another and will take approximately the following course: Figures 2-5. Beginning with the wave line, their sequence is similar in its effect to a charioteer trying to control his bolting horses. What further forms are then chosen depends on how the child has managed to control the movements of the previous exercises.

Before we continue I would like to answer a question that is surely arising: "Why a wave line?" One will get the answer by observing the children at work. Looking like strung together, bisected lemniscates the wave line's form element consists alternately of a swelling and a sucking principle. From the student's view, one arch bends outward and one comes to meet him. (Figure 52.) There is a threshold he must cross each time in his drawing—at the meeting point between the two semicircles. Two form principles are involved in the wave line which consists of semicircles joined in a row. As we are drawing a semicircle our arm is in a rotating, swinging motion; this is easily done by everyone. A circling movement is the toddler's first expression in drawing with a crayon. However, after drawing the first semicircle from left to right the circling tendency is interrupted and instead of a reverse movement that would be needed for the lower half of that circle the linear tendency is followed from left to right, leading to the lower half of a second circle. Unlike the first half, the center of which lies within the student, this second semicircle has its center outside, thus it is an arch that is pushing against him. Many children whose trouble is a lack of harmony between finding a center within themselves and being able to relate to the outside world get into considerable difficulty at this point of the drawing. They demonstrate how, for a lemniscate or a wave line to be shaped, certain preliminary conditions of soul and spirit are needed. It has even happened with fourteen-year-olds that the semicircle pushing against them was avoided in the following manner: Figure 5A. Every eurythmist knows the children who will run an ellipse instead of a lemniscate. Some children will construct a lemniscate by joining two circles. More advanced children will draw a figure three next to a mirrored three. Such things show that they have to reach a certain stage of development to be able to cross the threshold of a lemniscate or wave line where it moves from the inner to the outer form tendency.

One meets the threshold character of the wave line everywhere. If we associate contraction with the form principle of a circle as part of the wave line, and expansion with the linear direction from left to right, we discover the law of metamorphosis in the same way that we discover it everywhere in nature as contraction and expansion. A wave line consisting of semicircles which in a one-sided upward way develop the contracting element leads to a circle and finally to a point. Developing the wave line downward and following the principle of expansion leads to the straight line which is part of a circle of infinite dimension. (Figure 51.) If the wave line with which we started consists of geometrically exact semicircles, it is a threshold between circle and straight line—or, if we carry this further, a threshold between point and an infinite straight line which again will develop into a circle.

The form principle of the bisected lemniscate first appears in history during the Greek culture—beginning with stone slabs of early Cretan times. In relation to its threshold character Rudolf Steiner pointed first to the deep meaning of the Ionic column. In the fifth lecture of his *Curative Education* he developed a picture of man by illustrating our fourfold being with lemniscates that intersect in the chest, thus dividing the upper and lower part first of all into a head and a metabolic limb system. The intersection of the lemniscate is at the point where the head system (and with it our day consciousness), illustrated by the upper circle, merges into the metabolic limb system, illustrated by the lower circle. This crossing point is again the point where two different realms of consciousness meet: sleeping and waking. It is the realm of the rhythmic system, where dreams live. If we imagine a lemniscate in our face with our eyes as the centers of the two circles, the intersection will be at the point where our spatial consciousness focuses. Many of our children indicate by the position of their eyes that something is not as it ought to be. Some adults are also not always capable of maintaining the normal position of their eyes. Man is hemmed in by sense impressions that create the boundary for his experience of the outside world. If he would break through it with expanded consciousness he would not have the experience of an enlarged circle but of entering through a crossing point into supersensible space; there would be a lemniscate with the crossing point as threshold to an expanded experience. Rudolf Steiner pointed to the fact that at this very spot in man the clairvoyant observes lemniscates as light phenomena, which are open in the part facing the cosmos. In outer nature we find the bisected lemniscate in the wave, that is, on the surface where the boundary lies between two elements. Architecture is full of examples in which the varied use of the bisected lemniscate illustrates the level of consciousness of any historical time.

In our exercises we have the children draw the forms deriving from the wave line as a continuous band. Why? Whoever has practiced this kind of drawing with children knows how much will-force is engaged when a child does not simply draw a short form but is obliged to carry it through a whole continuous line. A very special force is being called forth and strengthened which does not even exist at first in children who are in curative homes. The intellect, which always wants something new, is now excluded in favor of a rhythmic movement, which through repetition works strongly on the will, and which is in itself beautiful. One does not let the child give way to his desire for free drawing but gives him specific tasks which he must carry out with great precision. In so doing he thereby experiences the archetypal laws underlying all forms. Contrary to the popular belief that every child is an artist and must not be influenced, the fact is that a child only accomplishes what he does—often charming achievements—through the influence of prenatal forces that still reach into his early years and then gradually fade away. Every art teacher knows from experience that as a child's intellect develops these abilities disappear without leaving a trace and that at this time the child is left empty-handed. Rudolf Steiner has pointed out to Waldorf School teachers that they should develop in the children a feeling for form in the realm of art before their urge to copy the outer world has awakened. Already at the very beginning of their school life the children are led to experience the curved and the straight line, angles and arcs, etc. Letters of the alphabet are made to emerge for them out of pictures. Beginning with their eighth year form drawing is increasingly practiced. For in their ninth year, when their imagination and thinking begin to separate (which until then had been completely united), the children feel an urge to represent the newly experienced outside world exactly, that is, in a naturalistic way. Naturalism is forced upon them more and more by their growing intellect. They will be protected against it if up to their tenth year they have practiced pure

form as art in its own right and if also they have experienced the character of colors in artistic work without having to portray natural objects.

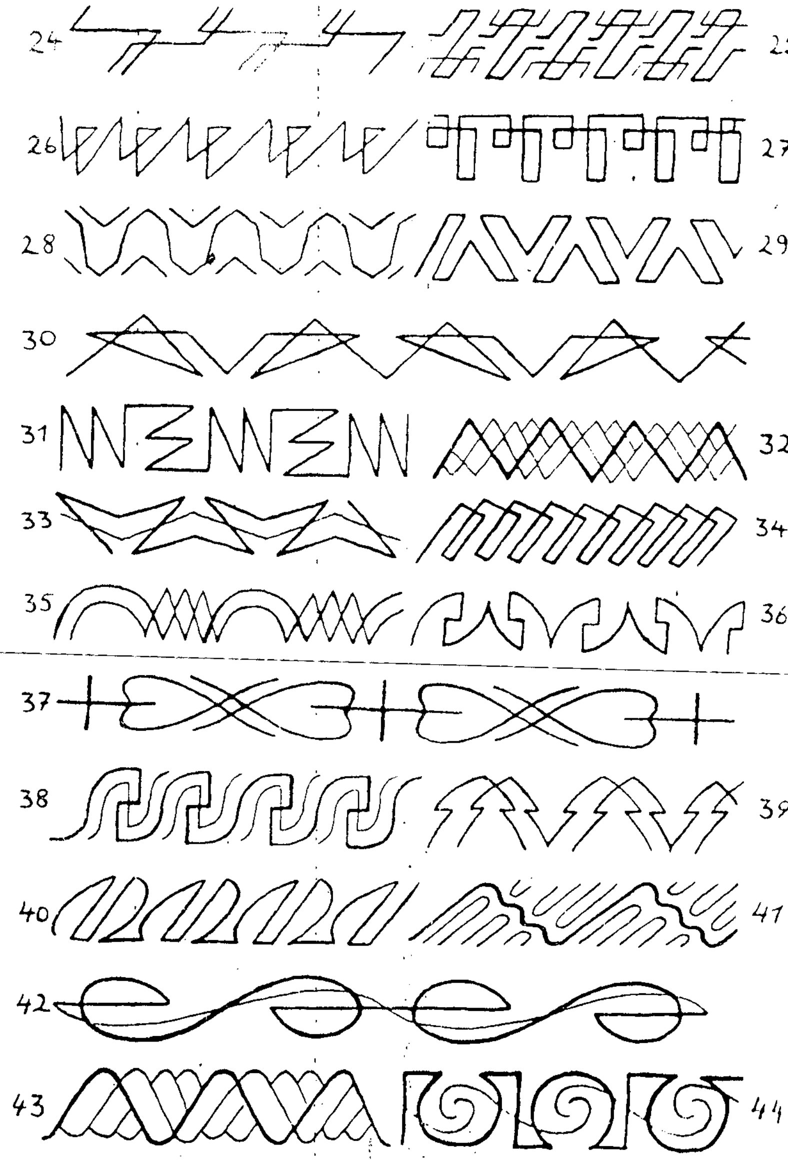
At this point one should say that in curative work one finds many exceptions to the rules that underlie the normal development of a child. A fourteen-year-old was incapable of copying the simplest nonobjective form. When he was given the task of drawing simple objects of everyday life which he knew and which came from his immediate surroundings he succeeded fairly well and in the drawings he used form elements which he had been incapable of drawing before. The impressions of forms from his surroundings had activated something in him that brought about this ability. He had not originally been capable of reproducing a pure form that was not related to an object. In this case we recognize the feeble-minded children: the children who, even though they have healthy eyes, are staring instead of looking, whose ego-force is not sufficiently involved in the seeing process. A fourteen-year-old of this group who was as yet unable to form a sentence, who could only utter single words, was also incapable of distinguishing a triangle from a square. After doing form drawing for three years, and having to recognize and draw ever more complex forms, he succeeded in drawing quite complicated ones. At the same time his speech developed, he learned to form whole sentences, his body developed dexterity and his thinking woke up. Although he had not learned a single letter up to his fourteenth year in spite of great efforts on the part of teachers and parents, it now became possible in the following years to teach him to read and write in a simple way.

Again and again Rudolf Steiner points to the fact that one should not teach calligraphy as a special subject, but rather the child should learn to draw beautiful lines, beautiful curves and angles, and artistic forms in general. Then he will have all the preparation he needs for beautiful handwriting.

A teacher must be wide-awake while working with children. By sensing and perceiving the inabilities of individual children he will know what his next step should be. That means that in a sense he is to be watching over a child's shoulder, following his movements, experiencing when the line halts, when the child's ability is breaking down due to his constitution. In this way the teacher is learning two things. For one thing, he will find a preparatory form that will help the child. Then, confirmed by many similar cases, he will realize that with that kind of constitution a child is not yet capable of coping with this specific form principle. In this way, out of the children's ability or lack of ability, the teacher is learning the relationship of the forms and their inherent laws to the human being. Very soon he will know what new form principle he can expect the children to draw, or whether for a while he ought to vary the forms the children have already learnt.

The great differences in the will-forces and in the imagination of various children soon lead a teacher to individual guidance. This enables him, while progressing slowly, to bring each child's impulses of movement with increasing sureness into controlled drawing. He is greatly helped by the children's joy in doing these exercises. With the group of feebleminded he must give most of his effort, to begin with, to stimulating their observation, finding it confirmed in their correct reproduction of the forms. With other children he must stress a clean and exact rendering. All the children who have difficulty incarnating are inclined to draw too quickly and inexactly. Hyperactive children usually dash off their forms with strong strokes. One often feels that the ego is not really involved, and the strangest skills are revealed. A certain craftiness shows up, which comes from the organ centered movements of the astral body. Telling the children to draw more slowly does not help. They need to be given forms in which the ego must predominate. Even drawing a symmetry is only possible with increased wakefulness. Also all forms requiring a reverse movement increase alertness and do not allow the child's attention to wander for a moment. (Figure 30.) All those forms which contain a movement that becomes increasingly more complex, are helpful in bringing the ego into action. Diagonal symmetries (Figures 8-11 etc.), intersecting forms, or forms that are not based on a geometric system and that are therefore difficult to follow, require the highest degree of wakefulness while drawing.

Children who are unable to let go of their mental images, who tend to have fixed ideas, often find it difficult to draw curved forms. (*Curative Education*, fifth lecture.) Again and again angles appear in their work—which points to excessive head involvement. It is a great help to give them free-swinging, round forms (Figures 13 and 20), also to plan extremely varied assignments for them so that they will constantly be confronted by new situations.



In the reverse situation, when mental images are diminishing and finally disappearing and memory is failing, one should draw on the board a form that is well adapted to the child but not too easily grasped; and have the child observe it carefully, then it should be rubbed out and the child required to draw it from memory.

Rudolf Steiner points to the fact that in general all drawing leads to dreaminess and has a strongly loosening effect. He recommends it as therapy, therefore, for children whose ego has been absorbed too strongly by the other members of their organism and who, in extreme cases, even show criminal tendencies. One could ask the question: what about the children who do not tend in this direction? Many years' experience shows that drawing brings about a great inner liveliness—which confirms Rudolf Steiner's indication. And yet one observes that the children manage to control this liveliness during their drawing. This is because the drawing has to do with forms, and one cannot cope with them unless one tries to establish harmony within oneself. Through this very effort the children actually enhance their illness and then learn to overcome it. Especially hyperactive children show breathless silence during their work. Again and again they remark that this is their favorite lesson, and there is almost always a strong attachment to the teacher who has been instrumental in bringing about their harmony. In order that a child's soul may thoroughly take hold and penetrate his physical body so that the process of incarnation is complete, an inner experience of the boundaries of his own body is necessary for the child, as Rudolf Steiner's spiritual science indicates. If he lives with his soul and spirit being at times too far out of his body, at times too far within it, the teacher must help by guiding him toward his actual contour. Once the child's attitude "I can't, I don't want to" has been overcome, the teacher can have the experience during the drawing that a moment comes when the child does feel himself to be within his skin. In the opposite case, when a child is actually prevented from establishing a proper relationship with his environment because his soul is blocked within one of his physical organs, dynamic drawing will help to start the process of incarnation going again. A perfect drawing is only possible in either case if a complete incarnation has been achieved by the child during his drawing. Even if after the drawing period the normality again disappears, the frequent achievement of such improvement in

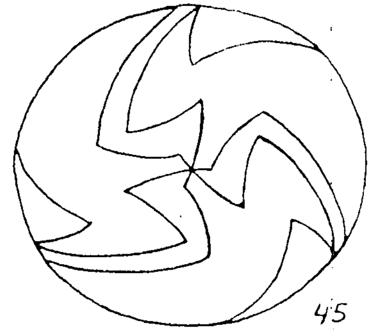
the child's life will lead eventually to norma[lity]

Once the children have gone beyond the wa... ... have drawn a sequence of metamorphoses (Figures 1-2..., it is good to develop similar forms out of the straight line (Figures 24-34), and later on to combine the two (Figure... 44). When we draw a curved form we feel how our slu... ing will becomes active. A straight form which again and again suddenly changes direction requires our fullest wakefulness at each turn if the drawing is to be beautiful. To children who live strongly in their intellect, those corners are a definite support for their inherent talent and they find it easy to manage them. There are also forms that are related to feeling. Working from the head down we have the straight forms; working upwards from the metabolic system we have the round forms. A combination of straight and round relates to the middle system. The teacher who has discovered this through his own practicing finds a large field of activity open to him. He will use these combinations in bands of rhythmically repeated forms, for they appeal to that part of the child's being which is predominant during the elementary school years.

After the band exercises one will introduce the rosette. It will be developed out of the circle, for which the children are permitted to use a compass. Any subdivision of it, however, should be drawn freehand in order to train the sense of balance. If the form motif is to be repeated in the spaces that result from subdivision it is important to draw it on the board in one space only and to have the children draw it into the other spaces within their circle without turning the paper. The less symmetrical the motif is, the more difficult will the task be. These exercises develop imagination very strongly. The task can gradually be made more complex. The movement of a second and third line, rhythmically repeated in each segment and adjusted to the existing forms, requires tremendous wakefulness. Through his comparing them with great care—one segment ought to correspond to the other—the child's sense of movement and of balance are strongly activated and trained. The child should be asked to draw simple motifs consisting of curved forms and then to change them into straight ones with angles. Guided by the teacher, he should invent simple exercises of metamorphosis himself, similar to Figures 62 and 63. Forms that are changing through expansion and contraction ought to be practiced.

Relationships of tension, as shown in Figures 52-61 and indicated by arrows, have as their starting point pure geometrical forms. As tension we mean here what is working as an interval between lines, forms or colors. The circle has a relationship of tension that is uniformly distributed since its radii meet the periphery at all points with equal length. It is the same with the arc as part of a circle. In case of a change by increasing the radius at one point, a relationship of tension occurs as indicated in Figures 54-57. The child must gradually become conscious of the invisible stress of forces within a drawing and learn to organize and control them. This requires the greatest alertness at every moment. What has already been created must be raised quickly into the realm of idea; then the child's imagination will be stimulated by it again and again and dominate further work.

Since all this is taking place in an artistic realm it is helping the child to develop his so-called lower senses: the sense of touch, the life sense, the sense of movement and of balance. If these four senses are not developed in the human artistic realm, if they are merely used in passive response to the demands of our technological world, a basis for their sound development will be missing. As we know, the small child is not yet capable of confronting his perceptions with concepts as an adult does. Concept and perception are to a certain degree still united. That is why his experiences, coming as they do from sense impressions, are so intense and why they even work formatively into his physical body. Our technological civilization forces extremely aggressive sense impressions upon the child which are alien to his soul life. They do severe damage. The teacher must create a counterbalance by providing many sense impressions for him out of a purely human sphere. Apart from the artistic activity the teacher's own mellow voice, a vivid pictorial manner of speaking, and the arrangement of the child's day into a rhythm that is quite the opposite of a motor beat, are things that should help to overcome the sense impressions deriving from technology. Our four lower senses need to unfold through sense perceptions deriving from human intercourse, not from technology. For they are the foundation upon which the sense of speech, sense of thought and sense of the ego have to be based. We know of the connection between movement and speech. Imagination that has been developed through artistic activity leads to thinking that goes beyond intellectual thinking. Sense perceptions that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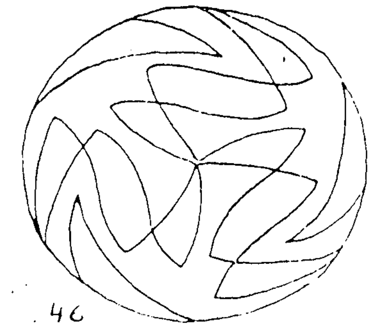
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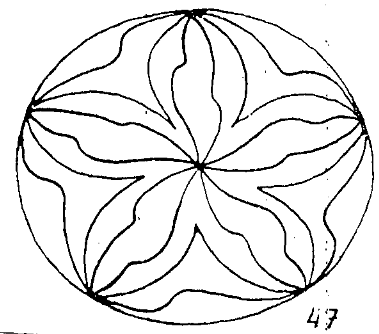
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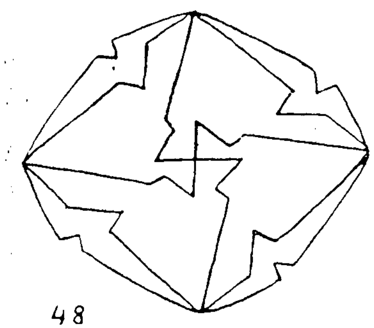
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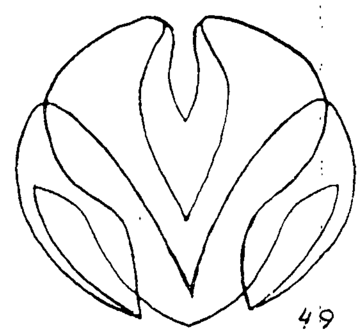
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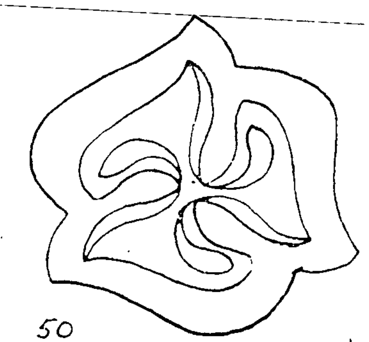
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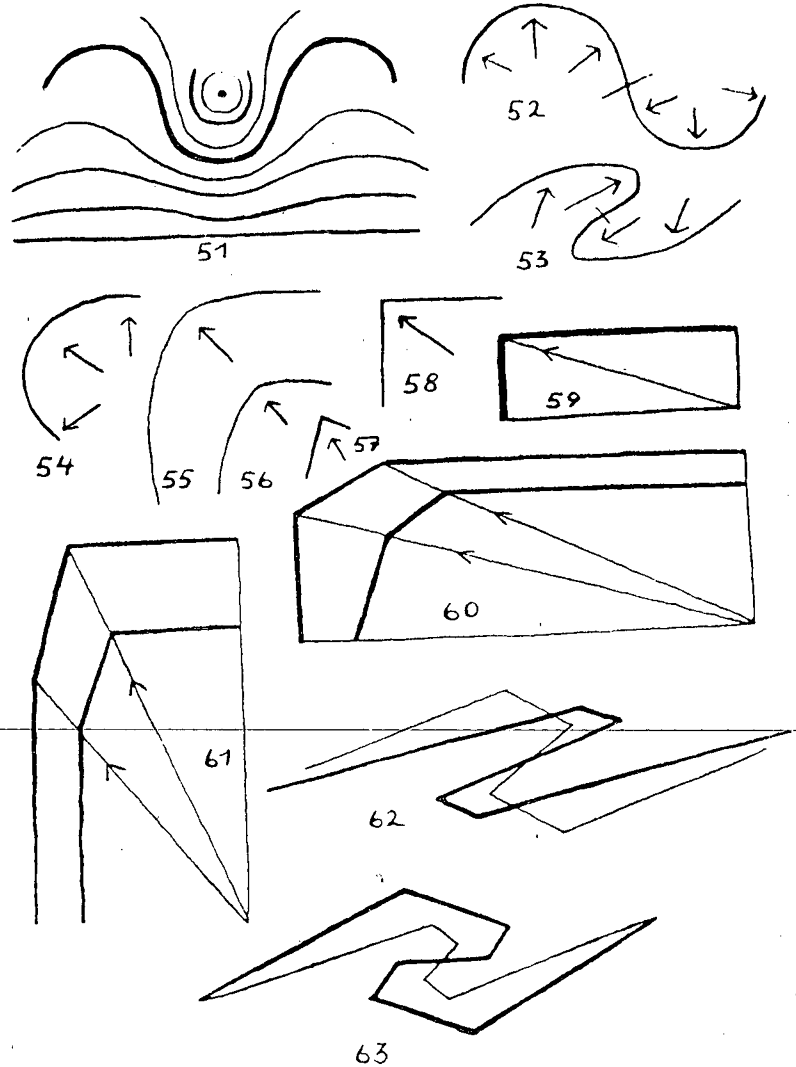
are trained mainly by synthetic impressions, such as cinema, record player, radio, television, are not a proper foundation for the ego sense. This sense provides a perception of the other person's ego and can only unfold through sense impressions of other human beings.

If the teacher feels that through a child's practice the artistic laws of form and harmony have become a part of his being, it is good to let him begin to work on his own. Forms from nature and geometrical forms should not be included in the assignments. At this point the result in most cases is that the child will bring to his work a wealth of ability and understanding. The practiced laws have become his property and are now his own individual language. Children's achievements differ greatly. While some child who has been drawing from his early years without guidance is now empty-handed, these children who have had all this training are now in command of creative wealth. They have not spent themselves; they have used the years for constructive learning. If a child has been left to create freely at too early an age his artistic output now will be like a child's playing on the violin who has not previously learned fingering or bowing. Something ill and frequently one-sided will be apparent in his work. The freely created forms, however, do point to the fact that the child has acquired the ability to lift himself again and again into a normal state while drawing. Looking together at his pictures when the lesson is over will calm the excitement that has arisen while he was drawing.

In developing the forms, beginning with the wave line, it is the teacher's task gradually to leave the realm of geometry that has had an extremely harmonizing effect upon the child and to introduce him to the world of living, dynamic forms. To imagine a geometrical circle as sun in a picture by Van Gogh is impossible; it will at once be felt as a foreign body. At this point divine and human ways separate. A strictly organized system is quite different from human creativeness that in every work of art finds its own laws freely and specifically provided for it. A child will bring intense seriousness to his own creative activity if previously he has practiced sufficiently. The creative process will frequently be interrupted; he will be looking at what he has done and considering it and, stimulated by it, his imagination will be finding highly individual solutions as he continues to work. The teacher witnessing this creativity feels deep reverence for the child's work and for the spirit being active in it. The article, *Image-Creating Powers of the Soul* (by Franz Loffler, in this book, untranslated), tells how a child fashioned an animal out of a chestnut shell and during a meal put it in front of him. Another child asked him why he did this. When the second child was asked what he thought it was, he replied: "Why, a chestnut shell." This story leads us to the concept of perception and imagination. Nowadays the latter is smothered in our children much too soon, partly by the intellectual environment provided them by adults, partly by inartistic teaching in our schools—schools which even for small children see their ideal in an accumulation of concepts and definitions. With affected grownup wisdom and prematurely old faces the little ones are silent accusers of our civilisation—which does not even know how to make a toy for a child anymore. Imitating what fills the adults' daily life, it lets the children do the same with mechanics and electricity.

A class of children practicing some art can be divided into those who are more imaginative and those who are more observant. In our particular setting it is a help not to seat the children of the same type together. Those with imaginative talent will always push ahead into the unlimited realm of forms and will surprise one by what they undertake. Again and again one of them will find new artistic elements which he then adapts in his individual way and they then become the property of the whole class. Copying is here out of the question since in most cases this would be too difficult. But the impulse of movement basic to the form has been experienced by the others as they watched their classmate and appears again in their own work. In that way each class has its own character. It is not advisable with the children who tend more toward observation to let an object appear in their drawing. Only after they have had much practice will one be sure that their work does not fall to a rather banal, inartistic level. The imaginative children should be brought closer to the realm of observation. The teacher can do this by helping them to see picture content in their freely created forms. However, he should be on his guard against naturalistic, inartistic elements. The language of pure gesture ought to be taught and ought to determine the picture's content. In doing this it is important that the effect be intuitively creative instead of naturalistic. In Waldorf School pedagogy this approach to the child is encouraged by developing thinking in the lower grades by way of the will and the feeling. Lively im-



agination in dynamic drawing develops abilities for painting, woodwork and handwork. This kind of drawing begins to mobilize the metabolic limb system, but the activity itself is carried out by the rhythmic man who receives directions from the head. It is an activity that involves all three soul principles and harmonizes them. We have moved from creative chaos through man's middle system towards the ordered world of beauty. Therefore dynamic drawing is an important means of therapy, on a level with painting with pure color, black-and-white drawing and wood carving.

We dealt with exercises based on the band and on the rosette. Now we would like to draw attention to the dynamic line that with the help of rhythm is covering a whole surface. (The paper should be as large as possible for this work.)

A child who takes his crayon and divides his drawing paper in half is using the same force that is at work when an infant raises his head or when he stands up and finally walks. The single stages of this development are its visible turning points. This struggle for the vertical makes us aware of the dimensions above and below, left and right, front and back. Humanity as a whole had achieved a new step in its ego-development when during the Renaissance the laws of depth in space were discovered. Rudolf Steiner gave Waldorf teachers exercises in symmetry that stimulate ego-activity. This is the force that makes us walk in space without falling down. It constantly balances our left-and-rightsidedness. It makes us an upright human being in a physical and also a spiritual sense.

In a larger connection this activity corresponds to what our ego experiences in sleep as compared to our day consciousness, and to life after death in its mirroring of our physical life on earth. In the physical realm logical thinking seems to go forward, while the reverse happens as mirror image when we enter the realm of the etheric.

If we look at the human figures of Egyptian and Greek art we can see how the vertical is emphasized. Actually this vertical is contained in every piece of art. It is the artist's active creative ego that unconsciously—just as in walking—determines orientation in space.

Letting retarded children begin by tracing the vertical has repeatedly been confirmed to be successful. Then one can go on drawing parallel lines close to one another, thus creating a rhythmically organized surface. The first line's dynamic quality is spreading over this surface. The child's next experience should be learning to draw a horizontal line. Pointing to the starry sky above us and to the horizon around us will relate these two main directions to their cosmic significance.

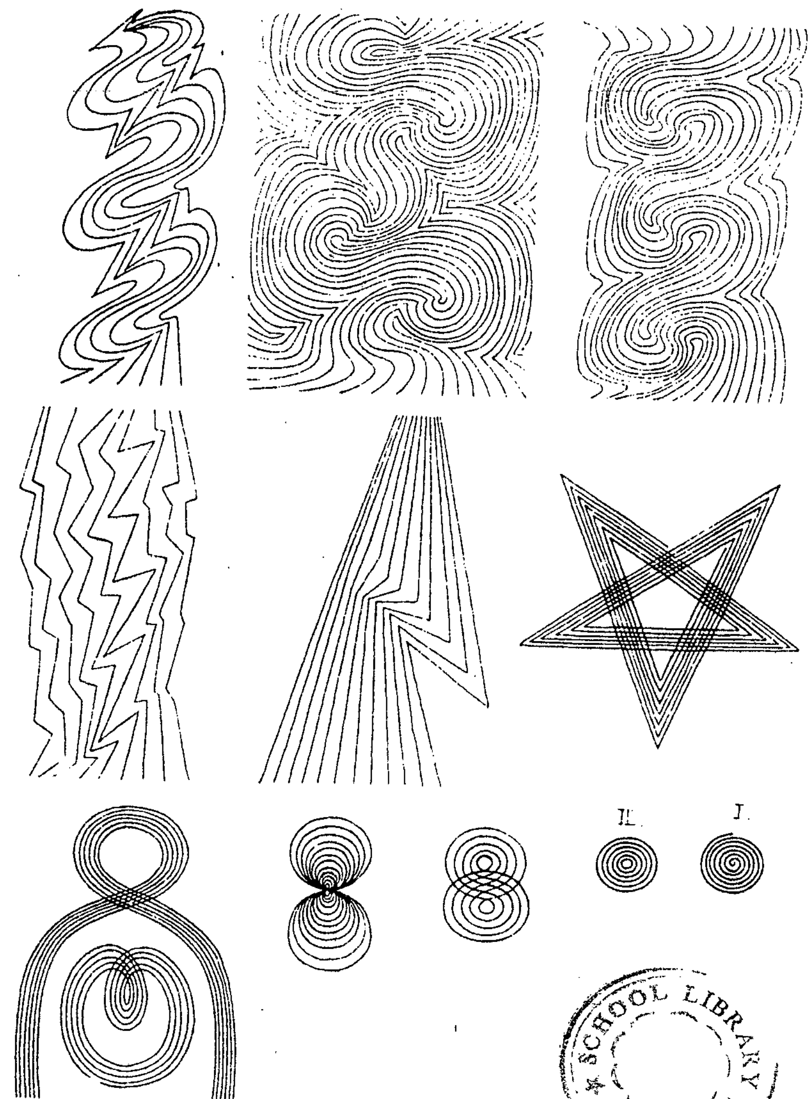
After working with the horizontal line in rhythmical patterns we can take up the streaming of the diagonal. The point of intersection can determine the beginning of a new activity. When we think of a straight line we are directing our consciousness to the shortest distance between two points, but when we form a spiral our attention is focused on the distance between the lines and on the developing curve. In this activity the ego is controlling the astral body.

For children who can only very slowly take hold of their metabolic limb system, for whom, therefore, the bringing of this system into movement is part of our therapy, drawing a spiral with its uninterrupted movement is very beneficial. (Picture I.)

For hyperactive children one should introduce rhythmically repeated interruptions. For instance, one will start with a point and draw a circle around it and then continue to draw more complete circles around the first ones. (Picture II.) A stone falling into water "draws" the same design. This points to a connection with the etheric. On the other hand Picture I. reminds us of a whirl of air, which points to a connection with the astral.

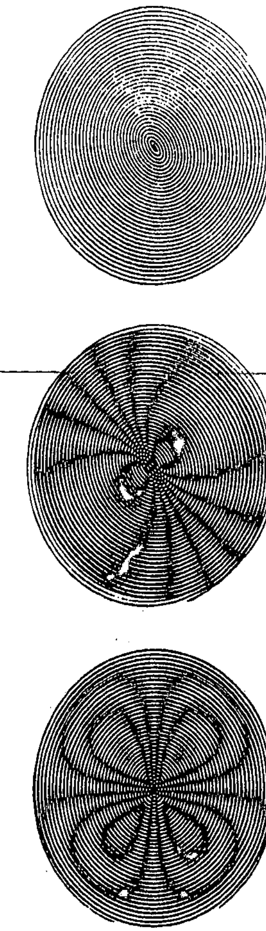
A breathing process is at work in an unwinding spiral that is leading over into an involving one. At the same time we are experiencing expansion and contraction as basic principles of life processes belonging to metamorphosis.

Further compositions with the straight line and the curve —with and without intersections—lead to a realm of unlimited possibilities. After sufficient practice of the flow of parallel lines, which is important as a beginning, one should go on by stressing the character of tensions belonging to intervals. Tensions come about when, because of an obstruction, lines take narrow distances from one another and then spread apart again when the area is free. (See drawing on last page.) The surface is moving into a flow without fixing individual forms by lines. Lumber from various kinds of trees shows how from year to year forms have come into being in a living process. Experiencing these forms in a room with wooden walls gives one a heightened feeling of well-being. Similarly while practicing with the child we can repeat a



moving line in various rhythmic ways and make it breathe. In this exercise the etheric and the astral are finding harmony in the activity of the ego.

HARMONOGRAPH



Unison (1:1): a spiral, a spiral drawn the same way over a spiral, and a spiral drawn the opposite way over a spiral.

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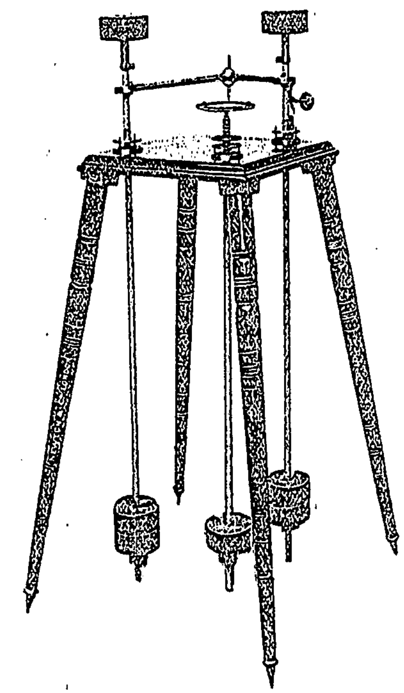
HARMONOGRAPH

A VISUAL GUIDE TO THE MATHEMATICS OF MUSIC



Anthony Ashton

WOODEN BOOKS

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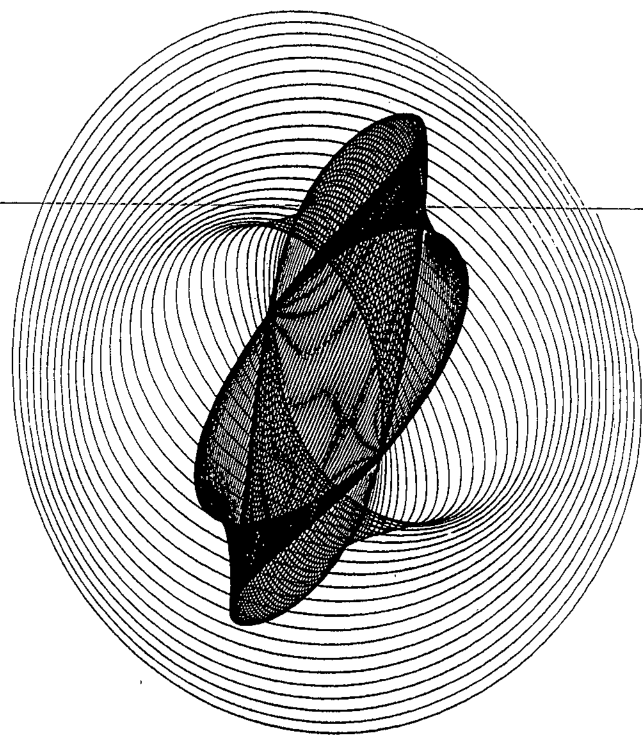
Dedicated to John, Antonia, and Imogen

Grateful acknowledgment to Harmonic Vibrations and Vibration Figures, *by Joseph Goold, Charles E. Benham, Richard Kerr, and L. R. Wilberforce, edited by Herbert C. Newton, Newton & Co., 1909;* Science and Music, *by Sir James Jeans, Cambridge, 1937;* Sound, *by John Tyndall, Appleton & Co., 1871;* Les Recreations Scientifiques, *by Gaston Tissandier, Masson, 1881. The image on page 47 is taken from the extraordinary book* Cymatics: A Study of Wave Phenomena and Vibration, *by Hans Jenny, © 2001, MACROmedia Publishing, and used here by kind permission. I am particularly grateful to my grandson for help with the intricacies of musical theory.*



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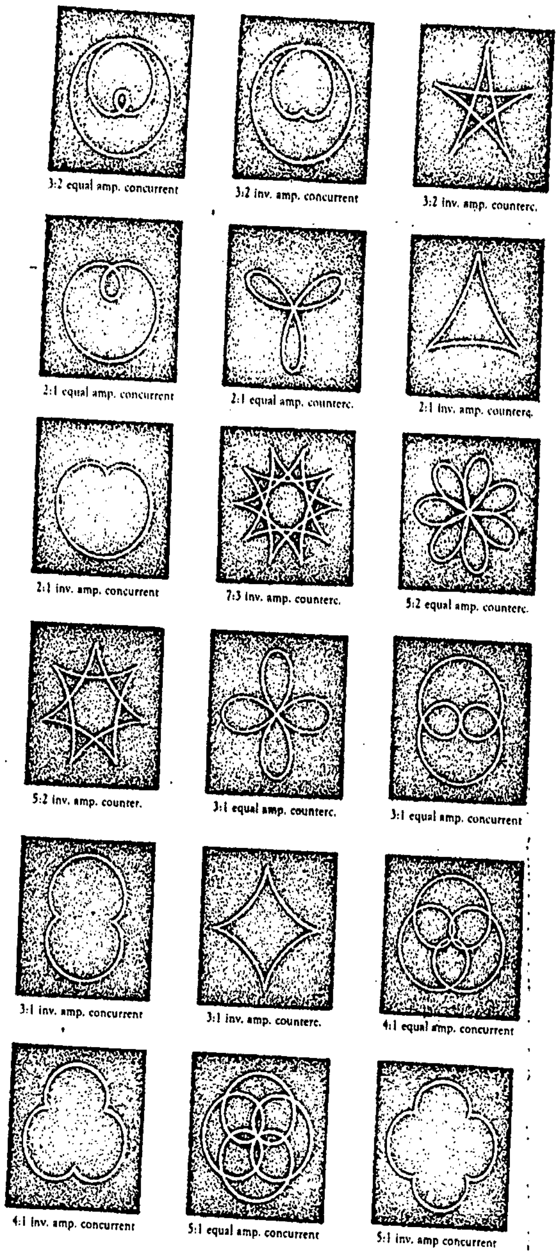


Harmonic patterns from Sir Thomas Bazley's Index to the Geometric Chuck *(1875), showing concurrent and countercurrent phases with equal and inverted amplitudes.*

INTRODUCTION

Many of the drawings in this book were produced by a simple scientific instrument known as a harmonograph, an invention attributed to a Professor Blackburn in 1844. Toward the end of the nineteenth century these instruments seem to have been in vogue. Victorian gentlemen and ladies would attend soirées or *conversaziones*, gathering around the instruments and exclaiming in wonder as they watched the beautiful and mysterious drawings appear. A shop in London sold portable models that could be folded into a case and taken to a party. There may well be some of these instruments hidden in attics all over the world.

From the moment I first saw drawings of this kind I was hooked. Not only because of their strange beauty, but because they seemed to have a meaning—a meaning that became clearer and deeper as I found out how to make and operate a harmonograph. The instrument draws pictures of musical harmonies, linking sight and sound.

However, before going any further I feel I should issue a health warning. If you too are tempted to follow this path, beware! It is both fascinating and time-consuming.

I have acknowledged my debt to the book *Harmonic Vibrations*. It was coming across this book in a library soon after the end of the second world war that introduced me to the harmonograph. Seeing that the book had been published by a firm of scientific instrument makers on Wigmore Street I went one day to see if they were still there. They were, though reduced merely to making and selling projectors. I went into the shop and held up my library copy of the book for the elderly man behind the counter to see.

"Have you any copies of this book left?" I asked him.

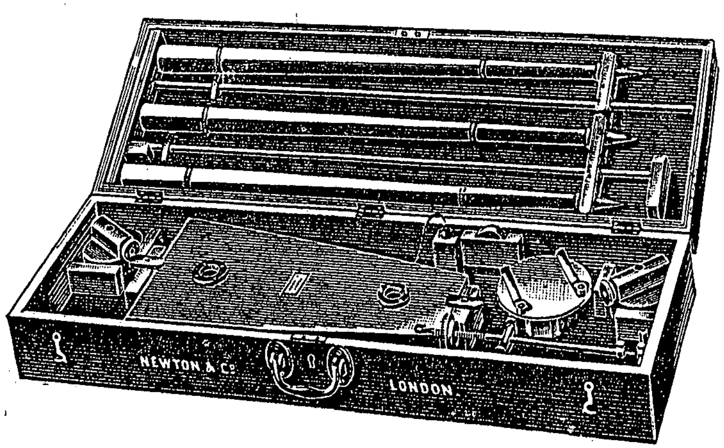
He stared at me as though I were some sort of ghost and shuffled away without a word, returning in a few minutes with a dusty, unbound copy of the book.

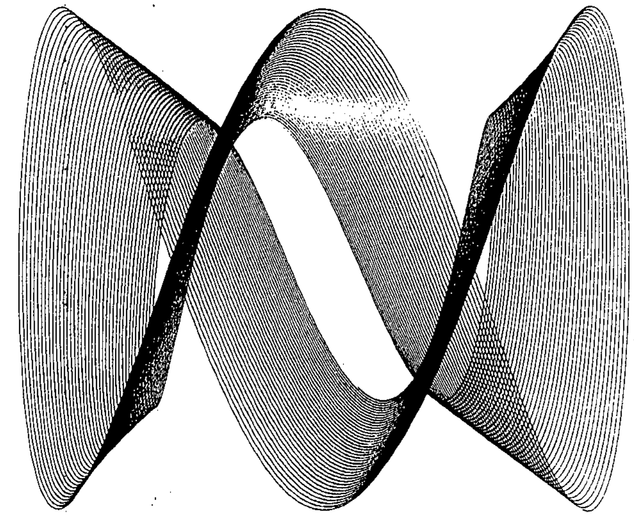
"That's marvelous," I said, "how much do you want for it?"

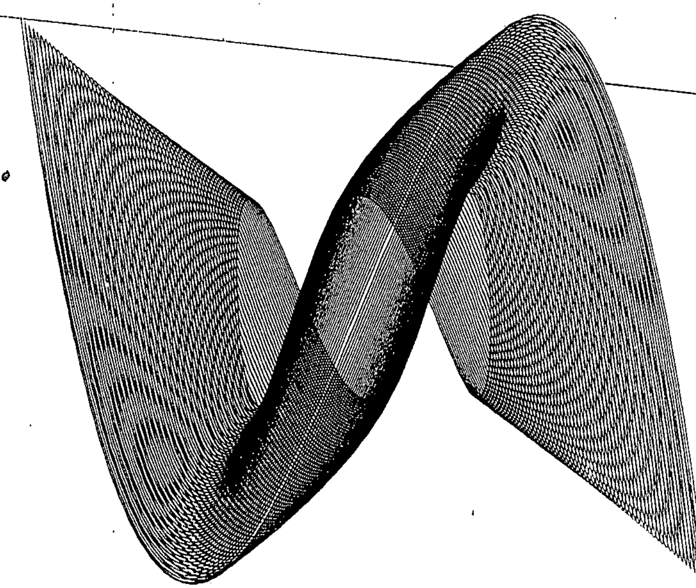
"Take it," he said, "it's our last copy, and we're closing down tomorrow."

So I have always felt that someday I must write this book.

Girton, 2002







The Discovery of Harmony

on passing a blacksmith

To understand what the harmonograph does we need first to glance at the elements of musical theory.

Pythagoras, some 2,500 years ago, is credited with discovering that the pleasing experience of musical harmony comes when the ratio of the frequencies consists of simple numbers. A tale tells how while taking a walk he passed a blacksmith's shop. Hearing familiar harmonies in the ringing tones of the hammers on the anvil, he went in and was able to determine it was the weights of the hammers that were responsible for the relative notes.

A hammer weighing half as much as another sounded a note twice as high: an *octave* (2:1). A pair weighing 3:2 sounded beautiful, a *fifth* apart. Simple ratios made appealing sounds.

The pictures opposite show experiments the philosopher went on to make (from Gafurio's *Theorica Musice*, 1492), as he found that all simple musical instruments work in much the same way, whether they are struck, plucked, or blown.

Deeply impressed by this link between music and number, Pythagoras drew the metaphysical conclusion that all nature consists of harmony arising from number, precursor to the modern physicist's assumption that nature conforms to laws expressed in mathematical form. Looking at the pictures you will see that in every example—hammers, bells, cups, weights, or pipes—the same numbers appear: 16, 12, 9, 8, 6, and 4. These numbers can be paired in quite a few ways, all of them pleasant to the ear, and, as we shall see, also pleasant to the eye.

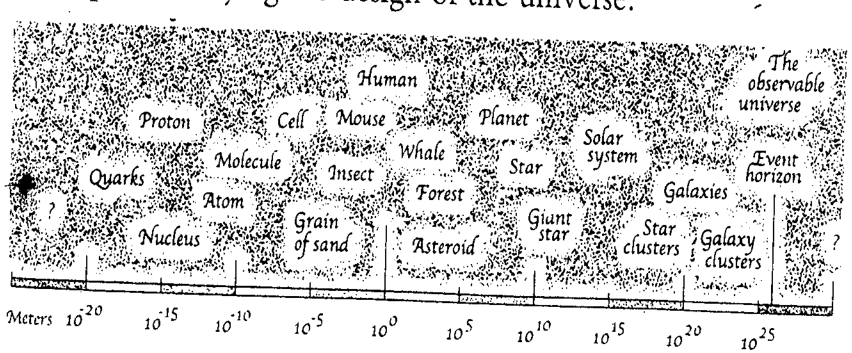
THE MONOCHORD OF CREATION

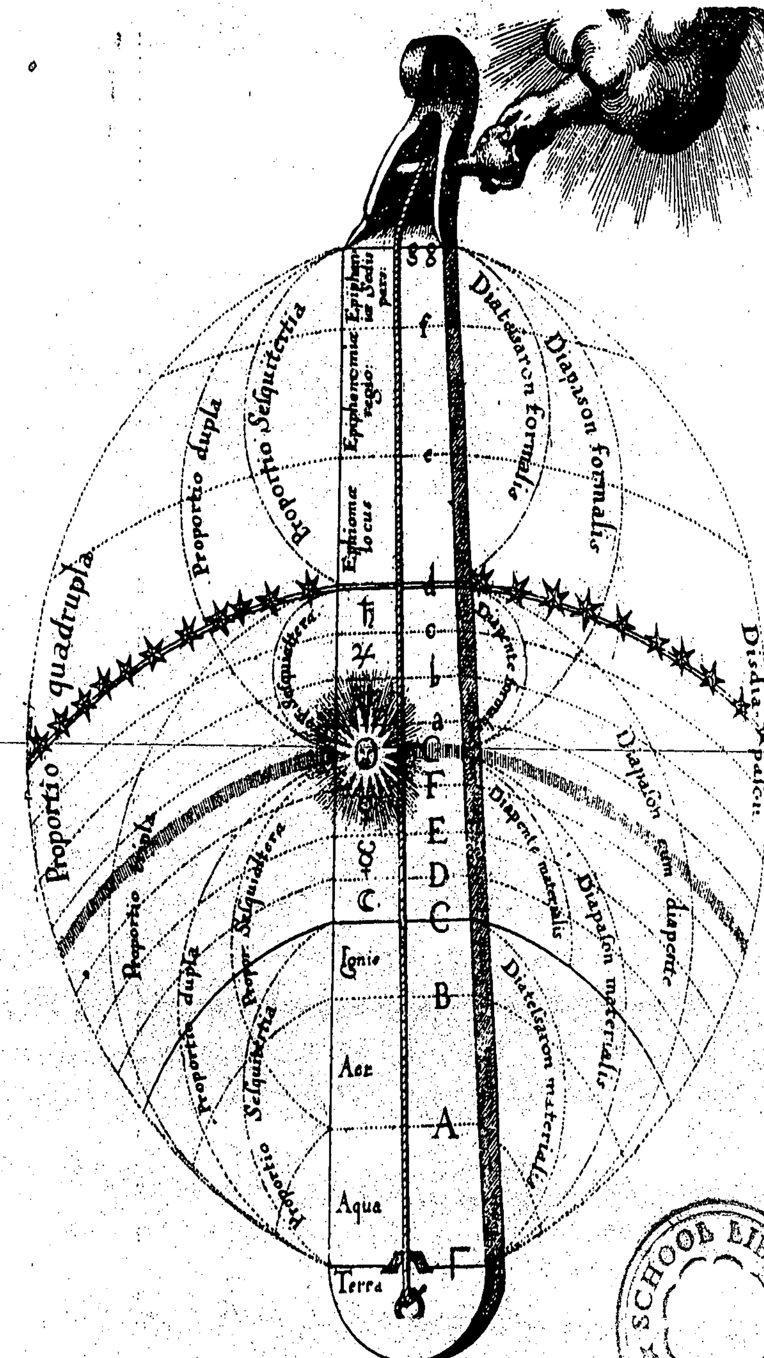
a singular string theory

There are seven octaves in the keyboard of a piano and nearly eleven in the total range of sound heard by the average person. The highest note of each octave has a frequency twice that of the first so the frequencies increase exponentially, on a scale beginning at 16 oscillations per second (16 *Hertz*) with the lowest organ note and ending with about 20,000 per second. Below 16 Hertz we experience rhythm. A range of ten octaves represents about a thousandfold increase in frequency ($2^{10} \approx 10^3$).

There is a hint here of what we can think of as the great monochord of the universe, also on a scale, this time stretching from a single quantum fluctuation at the bottom, to the observable universe at the top, passing through the various "octaves" of atom; molecule; quantities of solid, liquid, and gaseous matter; creatures great and small; planets; stars; and galaxies. Here too the scale is exponential, but usually measured in powers of ten, and covering a range of more than 10^{40}.

Robert Fludd's seventeenth-century engraving (*opposite*) tells a similar story: The musical scale follows the same exponential principle underlying the design of the universe.





Overtones and Intervals

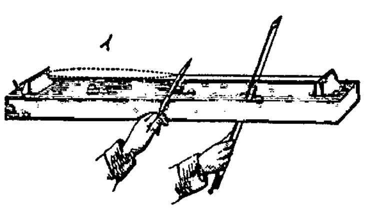
harmonic ratios in and outside the octave

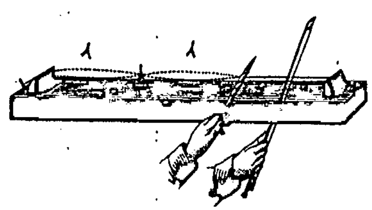
How are musical scales constructed? Listen very carefully as you pluck a string and you will hear not only the main note, or *tonic*, but also a multitude of other harmonics, the *overtones*.

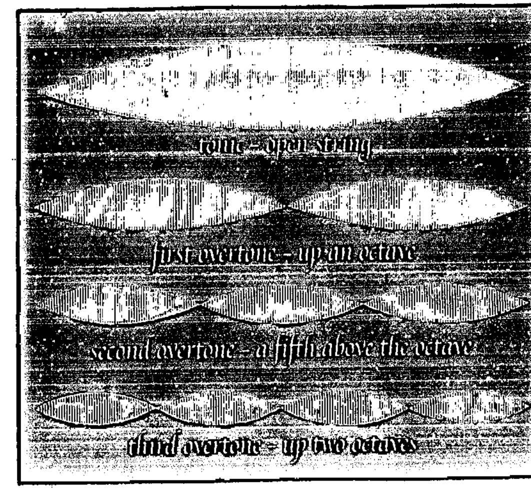
The principle is one of harmonic resonance, and affects not only strings and ringing hammers, but columns of air and plates too. Touching a string with a feather at the halfway or third point, as shown below, encourages regularly spaced stationary points, called *nodes*, and an overtone can be produced by bowing the shorter side. The first three overtones are shown opposite (*top*).

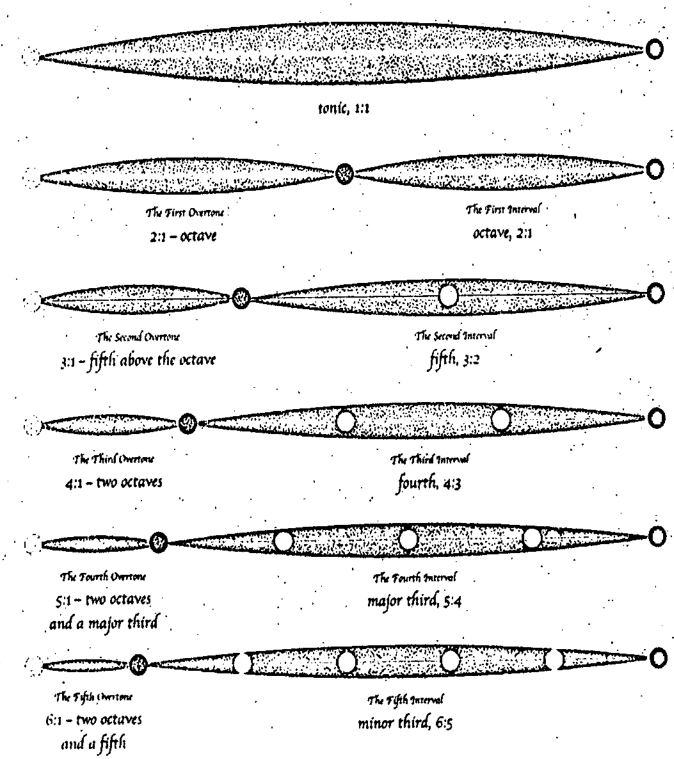
Musicians, however, need notes with intervals a little closer together than the overtone series, which harmonize within an octave. The lower diagram opposite shows the overtone series on the left, and the intervals developing within the octave on the right, in order of increasing dissonance, or complexity.

"All discord harmony not understood" wrote Alexander Pope. The brain seems to grasp easily the relationships implicit in simple harmonies, an achievement bringing pleasure; but with increasing complexity it falters and then fails, and failure is always unpleasant. For most people enjoyment fades as discord increases, toward the end of the series opposite. And, as we shall see, that is where the harmonograph drawings fade too.









HALFTONES

the fifth and the octave get their names

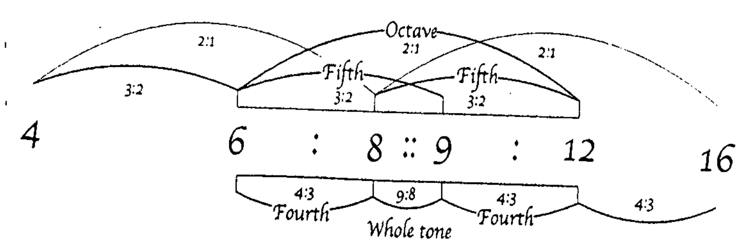
Pythagoras's hammers hide a set of relationships dominated by octaves (2:1), fifths (3:2), and fourths (4:3). The fifth and fourth combine to make an octave (3:2 x 4:3 = 2:1), and the difference between them (3:2 ÷ 4:3) is called a *whole tone*, value 9:8.

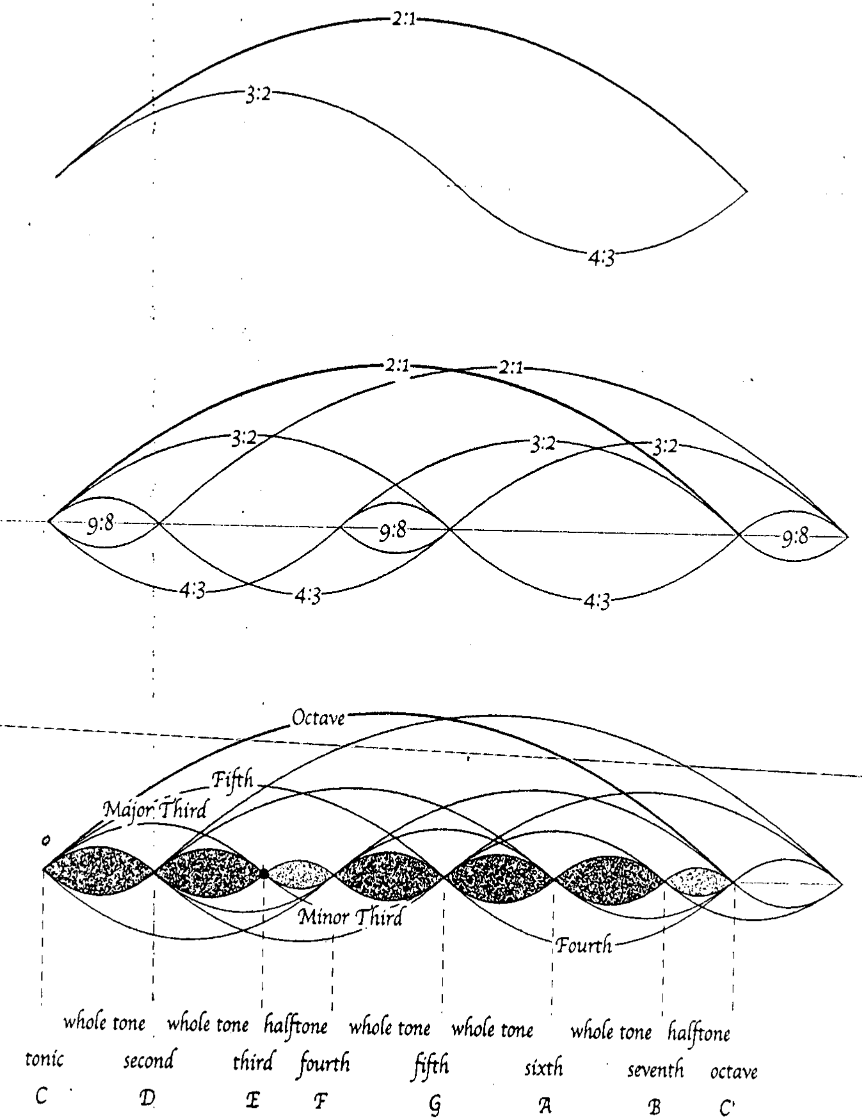
A natural pattern quickly evolves, producing seven discrete nodes (or notes), separated by two *halftones* and five whole tones, like the sun, moon, and five planets of the ancient world.

The fifth (3:2) naturally divides into a major third and minor third (3:2 = 5:4 x 6:5), the major third basically consisting of two whole tones, and the minor third of a whole tone and a halftone. The thirds can be placed major before minor (*to give the major scale shown in the third row, opposite*) or in other ways.

Depending on your harmonic moves, or *melody*, different *tunings* appear, for example two perfect whole tones (9:8 x 9:8 = 81:64) are not in fact the perfect major third 5:4, but are slightly sharp as 81:80 (the *syntonic* or *synoptic comma*, the Indian *sruti*, or *comma of Didymus*), which will be discussed more later.

Simple ratios, the octave and fifth, have given rise to a basic scale, a pattern of whole tones and halftones and, depending on where in the sequence you call home, seven *modes* are possible.





The basic manifestation of the scale. In Pythagorean tuning all whole tones are exactly 9:8, creating the leimma halftone of 256:243 between its major third (81:64) and the perfect fourth (4:3). The sixth and the seventh are defined as successive perfect whole tones above the fifth.

In Diatonic tuning the major third is perfect at 5:4, which squeezes the second whole tone to 10:9 (a minor whole tone), leaving 16:15 as the diatonic halftone up to the fourth. The diatonic sixth is 5:3, a major third above the fourth, a minor whole tone above the fifth. The diatonic seventh (15:8) is a major whole tone above that, a major third above the fifth and a halftone below the octave.

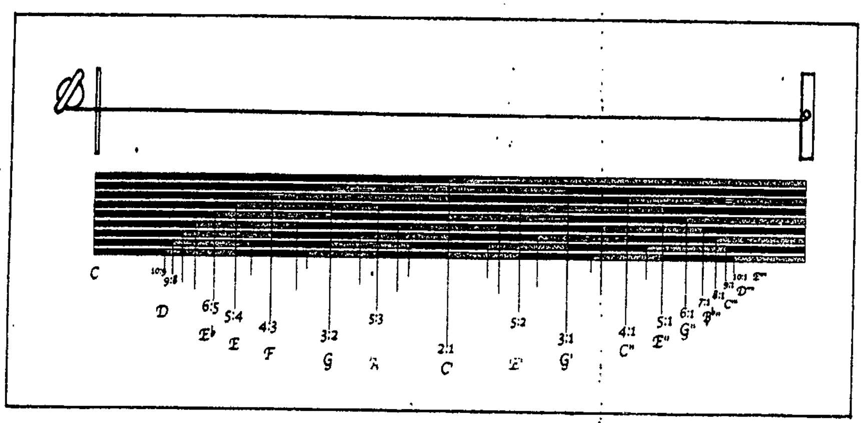
ARRANGING THE HARMONICS

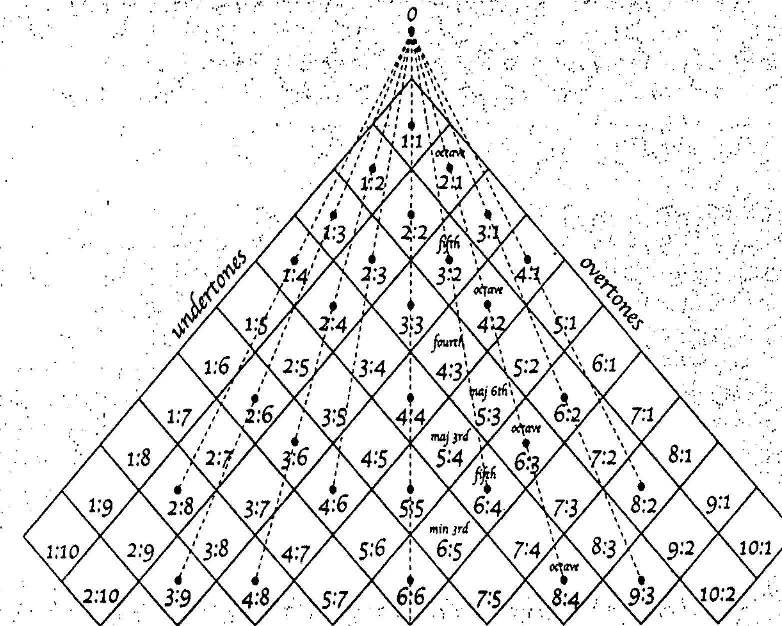
the power of silence

The simple ratios of the primary overtones and undertones can be plotted on an ancient grid known as a *lambdoma* (*opposite, top*), after the greek letter λ. Some intervals are the same (8:4 = 6:3 = 4:2 = 2:1), and if lines are drawn through these it quickly becomes apparent that the identities converge on the silent and mysterious ratio 0:0, which is outside the diagram.

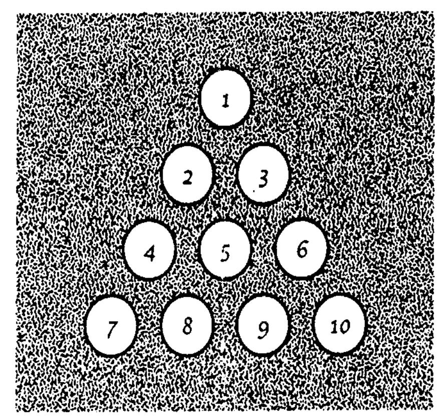
A further contemplative device used by the Pythagoreans was the *Tetraktys*, a triangle of ten elements arranged in four rows (1+2+3+4=10). The basic form is given opposite, lower left, the first three rows producing the simple intervals. In another lambdoma (*opposite, lower right*), numbers are doubled down the left side and tripled down the right, creating tones horizontally separated from their neighbors by perfect fifths. After the trinity (1, 2, and 3) notice the numbers produced, 4, 6, 8, 9, 12, and then look again at the picture on page 5.

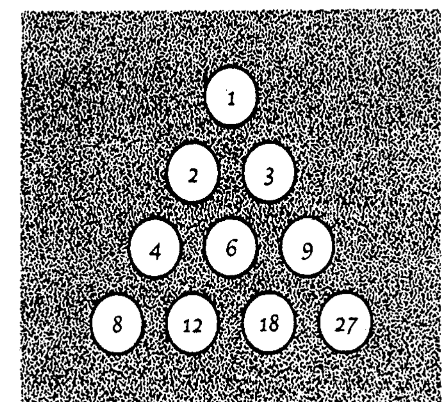
Below are interval positions on a monochord.





Pythagorean and medieval tunings, called 3-limit, recognized no true intervals except for ratios involving 1, 2, and 3. The lambdoma below, right expresses this numerically as any element relates to any neighbor by ratios only involving 1, 2, and 3, so we can move around by octaves and fifths. Squares ($4=2^2$, $9=3^2$) and cubic volumes ($8=2^3$, $27=3^3$) also appear. Add further rows and the numbers for the Pythagorean scale soon appear—1 9:8 64:81 4:3 3:2 27:16 16:9 2:1. This has four fifths and five fourths but no perfect thirds or sixths. These came later with the diatonic scale and its perfect thirds (6:5:4) as polyphony and chords slowly took over from plainchant and drone.





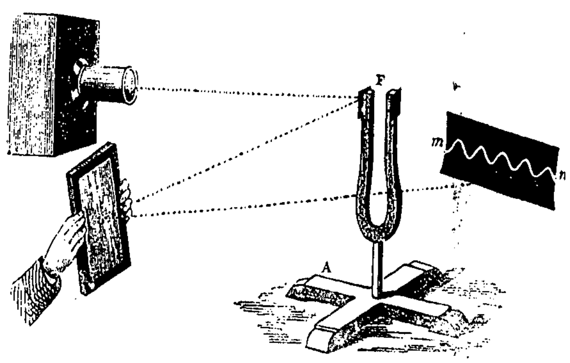
Lissajous Figures

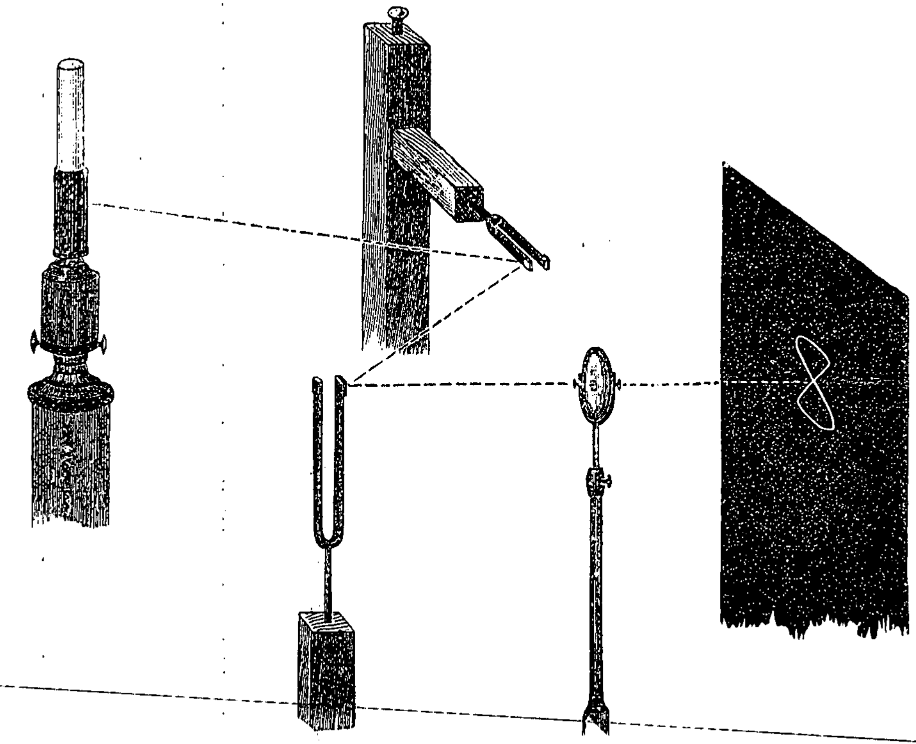
sound made shape

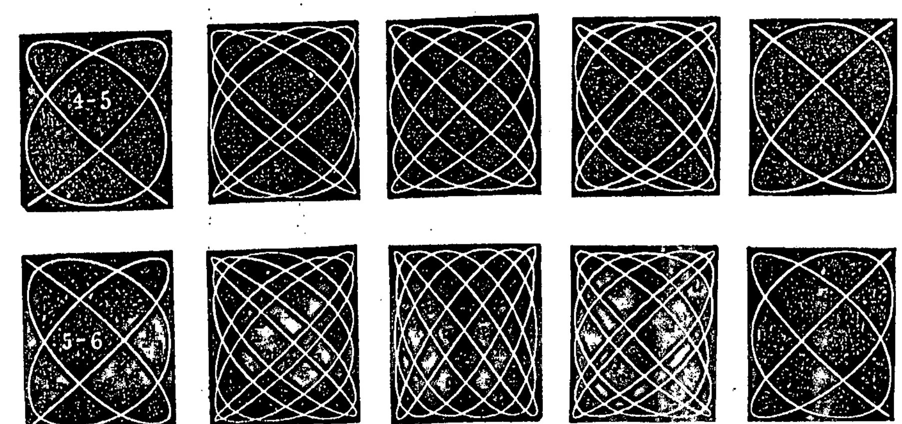
In the mid-nineteenth century, Jules Lissajous, a French mathematician, devised an experiment: He found that if a small mirror was placed at the tip of a tuning fork, and a light beam was aimed at it, the vibration could be thrown onto a dark screen. When the tuning fork was struck, a small vertical line was produced, and if quickly cast sideways with another mirror it produced a sine wave (*below*).

Lissajous wondered what would happen if instead of casting the wave sideways he were to place another tuning fork at right angles to the first to give the lateral motion. He found that tuning forks with relative frequencies in simple ratios produced beautiful shapes, now known as Lissajous figures.

On the screen (*opposite, top*), we see the octave (2:1) as a figure eight, and below it various *phases* of the major and minor third. These were some of the first fleeting pictures of harmony, which were doubtless familiar to Professor Blackburn when he devised the harmonograph.







THE PENDULUM

keeping time

A fundamental law of physics (in one formulation) states that left to itself any closed system will always change toward a state of equilibrium from which no further change is possible.

A pendulum is a good example. Pulled off center to start, it is in a state of extreme disequilibrium. Released, the momentum of its swing carries it through to nearly the same point on the other side. As it swings it loses energy in the form of heat from friction at the fulcrum and brushing against the air. Eventually the pendulum runs down, finally coming to rest in a state of equilibrium at the center of its swing.

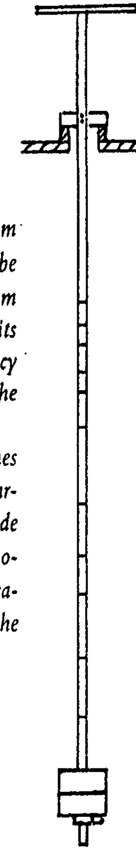
Going back 500 years, Galileo, watching a swinging lamp in the cathedral of Pisa, realized the frequency of a pendulum's beat depends on its length: The longer the pendulum the lower the frequency. So the frequency can be varied at will by fixing the weight at different heights. Most important, as the pendulum runs down, the frequency stays the same.

Here, therefore, is a perfect way to represent a musical tone, slowed down by a factor of about a thousand to the level of human visual perception. For a simple harmonograph two pendulums are used to represent a harmony, one with the weight kept at its lowest point, while the weight on the other is moved to wherever it will produce the required ratio.

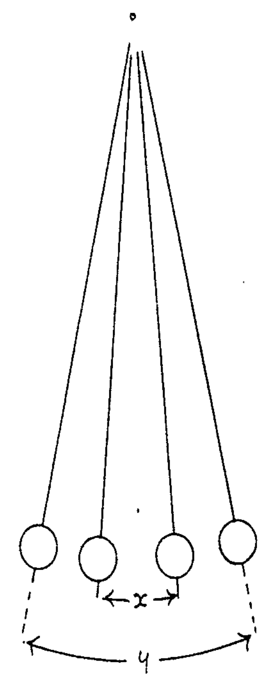
As we shall see, the harmonograph combines these two vibrations into a single drawing, just as two musical tones sounded together produce a single complex sound.

The theoretical length of the variable pendulum that will produce each harmony can be calculated, for the frequency of a pendulum varies inversely with the square root of its length. This means that while the frequency doubles within the octave, the length of the pendulum is reduced by a factor of four.

Figures are given for a pendulum 32 inches (80 cm.) long, a convenient length for a harmonograph. These theoretical markers provide useful "sighting shots" for most of the harmonies. Note that the pendulum length is measured from the fulcrum to the center of the weight.



Interval Name	Approx. Note	Diatonic Ratio	Length (cm)	Freq. (s^{1})
Octave	C'	2:1	20	66.0
Maj. 7th	B	15:8	22.8	62.8
Min. 7th	B'	9:5	24.7	59.4
Maj. 6th	A	5:3	28.8	55.8
Min. 6th	G'	8:5	31.2	53.6
5th	G	3:2	35.6	50.3
4th	F	4:3	45.0	44.7
Maj. 3rd	E	5:4	51.2	41.9
Min. 3rd	E'	6:5	55.6	40.2
2nd	D	9:8	63.2	37.7
Halftone	C'	16:15	70.3	35.7
Unison	C	1:1	80	33



When a pendulum is pulled back and then released, the weight tries to fall toward the center of the Earth, accelerating as it does so. As the pendulum runs down, the rate of acceleration, and so the speed of travel, is reduced, but in equal proportion to the distance of travel.

The result is that the period (the time taken for two beats) or the number of periods in a given unit of time (the frequency) remains unchanged. In the picture to the left the frequencies of beats x and y are the same.

For the pendulum formula, see page 53.

Two Harmonographs

lateral and rotary

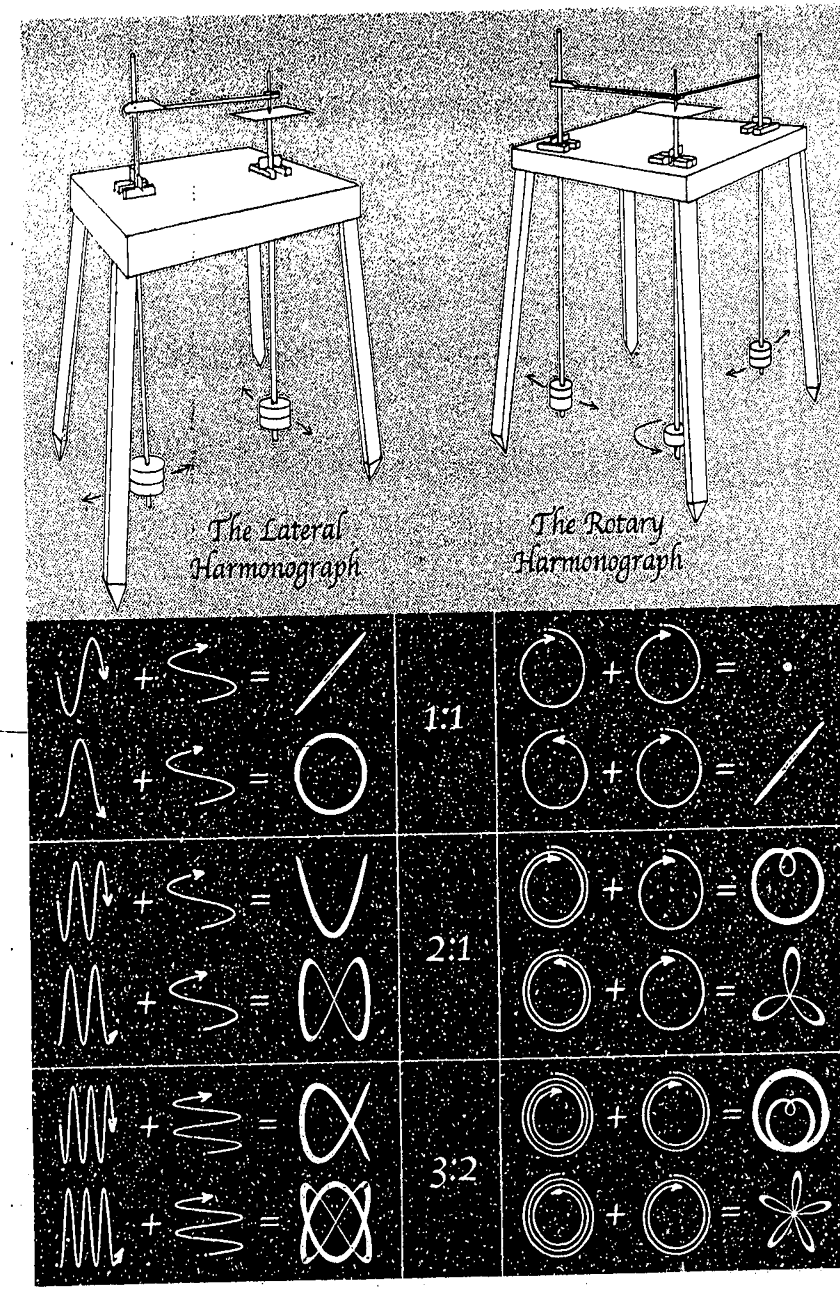
In the simplest version of the harmonograph two pendulums are suspended through holes in a table, swinging at right angles to one another. Projected above the table, the shaft of one pendulum carries a platform with a piece of paper clipped to it, while the shaft of the other pendulum carries an arm with a pen.

As the pendulums swing, the pen makes a drawing that is the result of their combined motion (*opposite, left side*). Both pendulums begin with the same length, one is then shortened by sliding the weight upward and securing it with a clamp at various points. The harmonic ratios can be displayed in turn.

By using three pendulums, however, two circular, or rotary, movements can be combined, with fascinating results (*opposite, right side*). Two of the pendulums swing at right angles as before, but are now both connected by arms to the pen, which in all rotary designs describes a simple circle.

Situated under the circling pen, the third and variable pendulum is mounted on gimbals, a device familiar to anyone who has had to use a compass or cooking stove at sea. Here it acts as a rotary bearing, enabling the pendulum carrying the table to swing in a second circle under the pen. As the pen is lowered the two circles are combined on the paper.

A further source of variation is also introduced here, for the two circular motions can swing in the same (concurrent) or opposite (countercurrent) directions, producing drawings with very different characteristics.



Two harmonographs and some of the simple patterns they draw. On the left the simple lateral version and its patterns (open and closed phase); on the right the three-pendulum, rotary harmonograph and its drawings (concurrent and countercurrent).

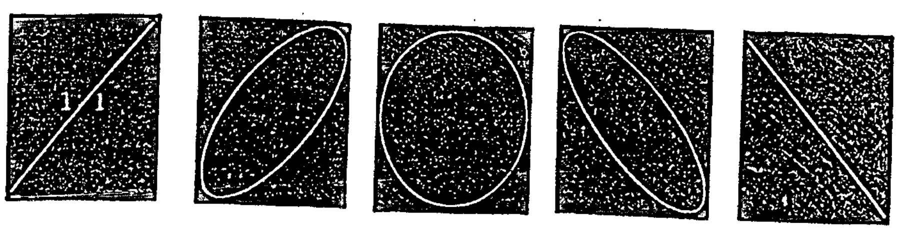
SIMPLE UNISON—1:1

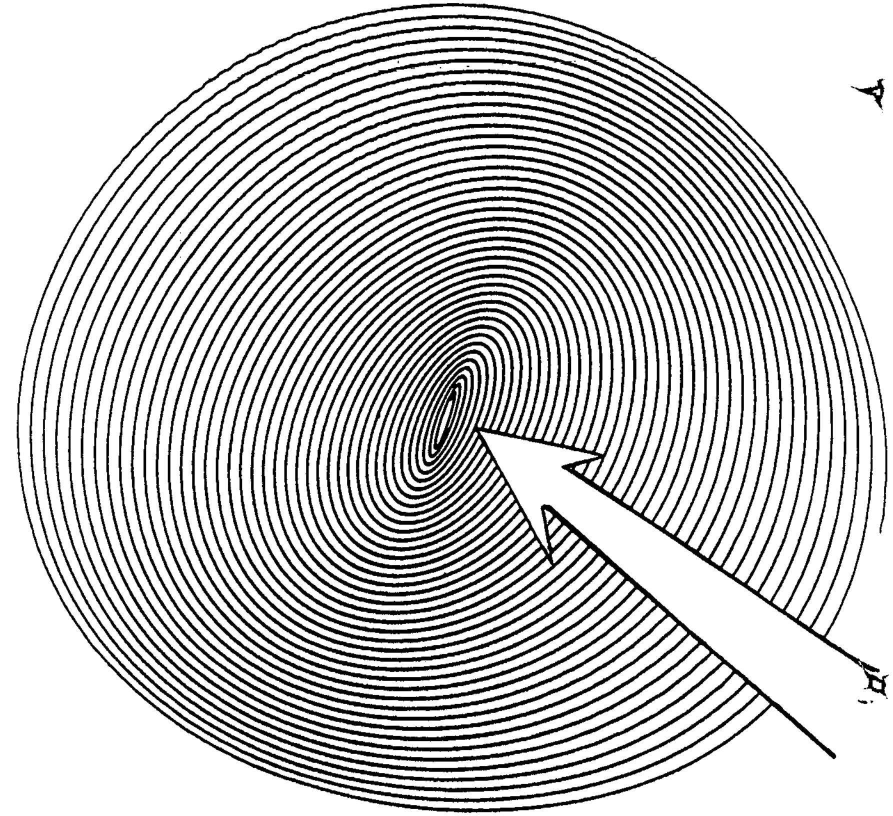
and the arrow of time

The simplest harmonograph drawing is produced when both pendulums are the same length and the table is stationary. With the pen held off the paper both pendulums are pulled back to their highest points. One is released, followed by the other when the first is at its midpoint. The pen is then lowered onto the paper to produce a circle developing into a single spiral.

If the two pendulums are released together then the result will be a straight diagonal line across the paper, the "closed" phase of the harmony, as opposed to the circular "open" phase. At intermediate phase points elliptical forms appear (*below*).

The running-down of harmonograph pendulums is an exact parallel to the fading of musical notes produced by plucked strings, and can also be thought of as graphically representing the "arrow of time" (*opposite*), with the unchanging ratios of the frequencies representing the eternal character of natural law. The characteristics of the drawings result from the meeting of the running-down process with the laws represented by the various frequency ratios. We see that music, like the world, is formed from unchanging mathematical principles deployed in time, creating complexity, variety, and beauty.





The inexorable direction of change, linked to the asymmetry of time (before-now-after), was vividly described by the scientist Arthur Eddington (1882–1944) as "the arrow of time." Throughout the process of continuing universal degradation, the dwindling stock of useful energy encounters a hierarchy of fixed physical laws conforming to mathematical formulas, and from the interaction of these unchanging laws with the arrow of time comes a changing world of astonishing complexity, variety, and beauty. The pendulum runs down from a state of disequilibrium to one of equilibrium, and the same is true, we are told, of the universe, the ultimate closed system. From a state of extreme disequilibrium it plunged via the Big Bang toward its future ultimate state of utterly dark, frozen equilibrium. Between the beginning and the end there is a continual, cumulative transformation of useful energy, capable of forming temporary structures and causing events, into useless energy forever lost.

NEAR UNISON

lateral phases and beat frequencies

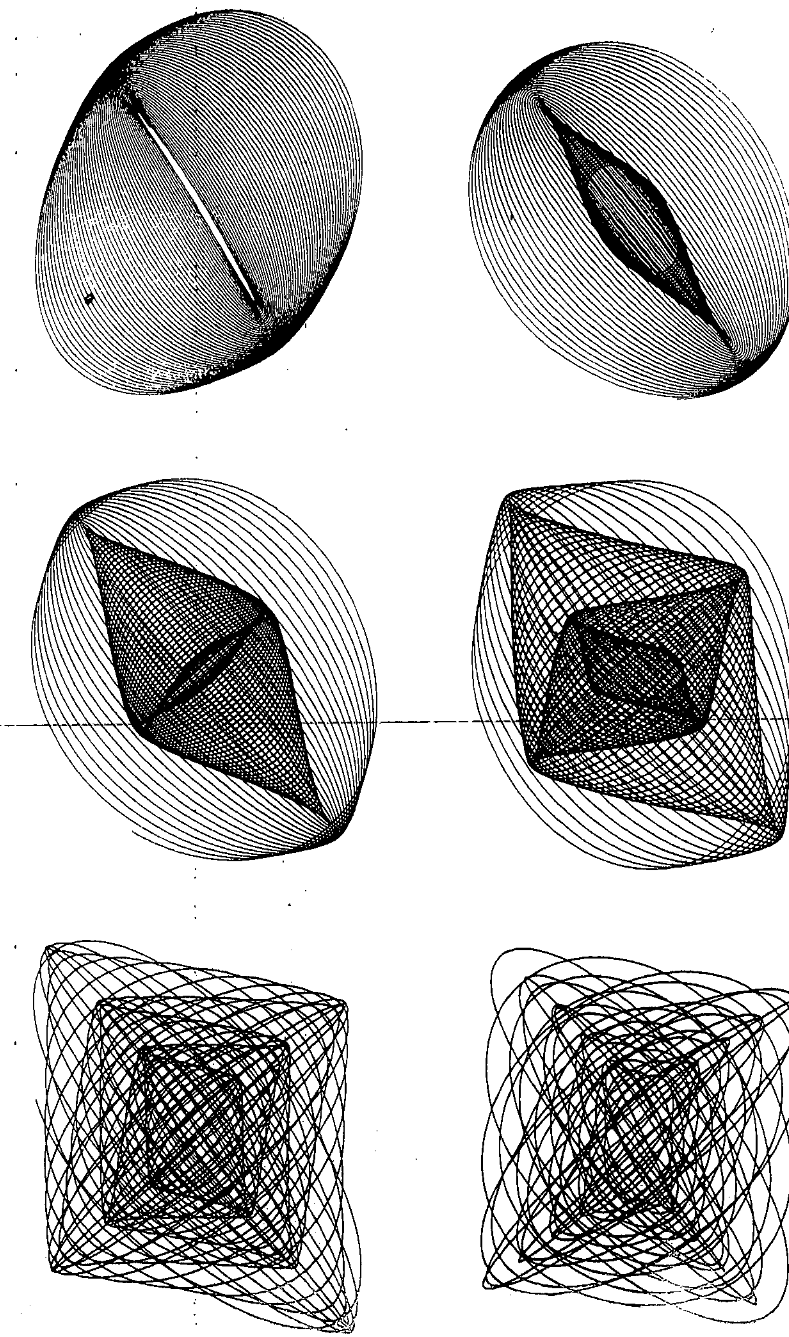
A source of pleasing variety in harmonograph drawings comes from small departures from perfect harmonies. This seems to involve a principle widespread in nature as well as in the work of many artists. There is a particular charm in the near miss.

An example from music suggests itself here. When two notes are sounded in near unison, the slight difference in their frequencies can often add richness or character to the sound. The two reeds producing a single note in a piano accordian have slightly different frequencies, the small departure from unison causing *beats*, a warbling or throbbing sound (*see page 53*).

Set the weights for unison and then shorten the variable pendulum slightly. Swing the pendulums in open phase, producing a circle turning into an increasingly narrow ellipse and then a line. If the pen is allowed to continue, the line will change into a widening ellipse, a circle, and a line again at right angles to the first. And so on. The instrument is working its way through the phases of unison shown on page 20.

If the variable pendulum is then further shortened in stages, a series of drawings like those opposite will be produced. The repetitive pattern represents beats with increasing frequency as the discrepancy between the notes widens. Eventually the series fades into a scribble that is a fair representation of discord, though even here there is a hint of some higher number pattern.

For most people this fading of visual harmony occurs at about the same point as the audible harmonies fade.



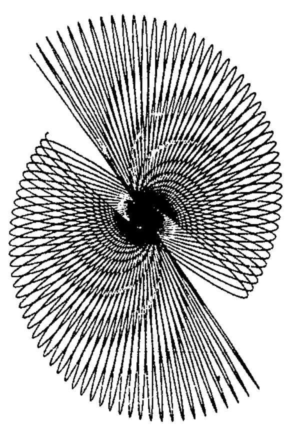
ROTARY UNISON—1:1

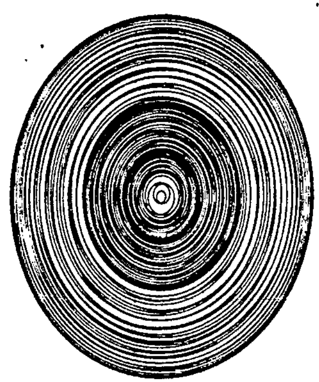
eggs and shells

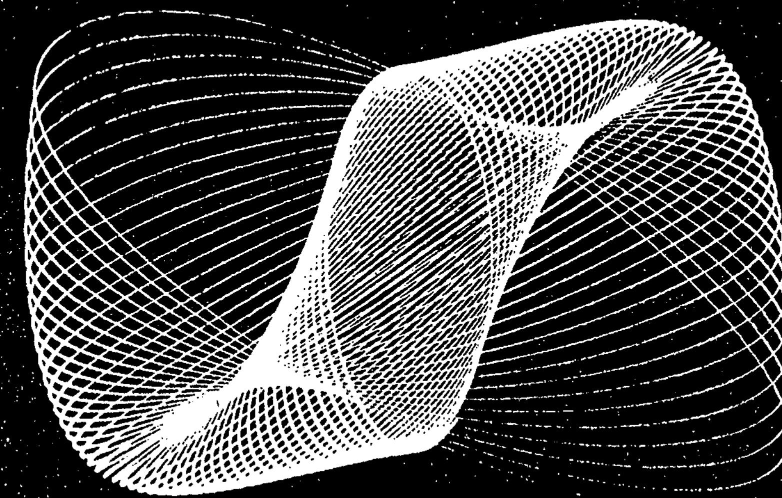
Unison in contrary motion produces a straight line across the paper, like the closed phase of lateral unison. From concurrent motion there comes a mere dot that turns into a line struggling toward the center, pen and paper going around together.

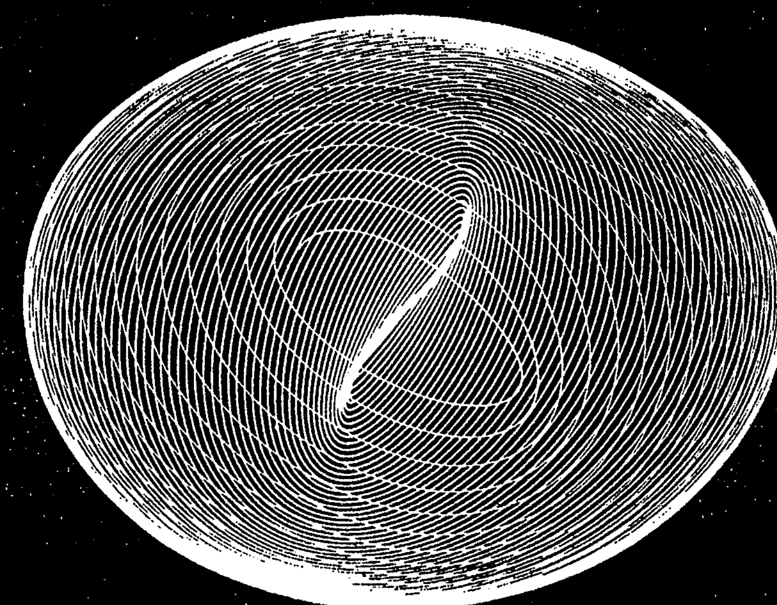
At first this is disappointing. However, changing to near unison is richly rewarding. In contrary motion come a variety of beautiful, often shell-like, forms with fine cross hatchings. For best results lift the pen off the paper well before the pendulums reach equilibrium.

Surprisingly, from concurrent near-miss motion there come various spherical or egg-shaped forms. To produce an egg shape the pen should be lowered when it is dawdling at the center. It then spirals its way outwards, reaching a limit before returning as the pendulums run down. Because the lines toward the perimeter get closer together, the drawing appears three dimensional.









The Lateral Octave—2:1

figure eights and wings

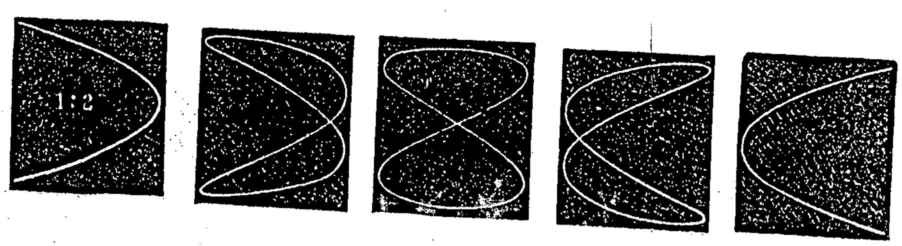
After unison the next harmony to try is the octave. Here there is a technical difficulty: The variable pendulum has to be very short, and because of the greater amount of friction involved it runs down quickly. The trick is to add a weight to the top of the invariable pendulum, which slows it down (*see title page*). The variable pendulum can then be longer.

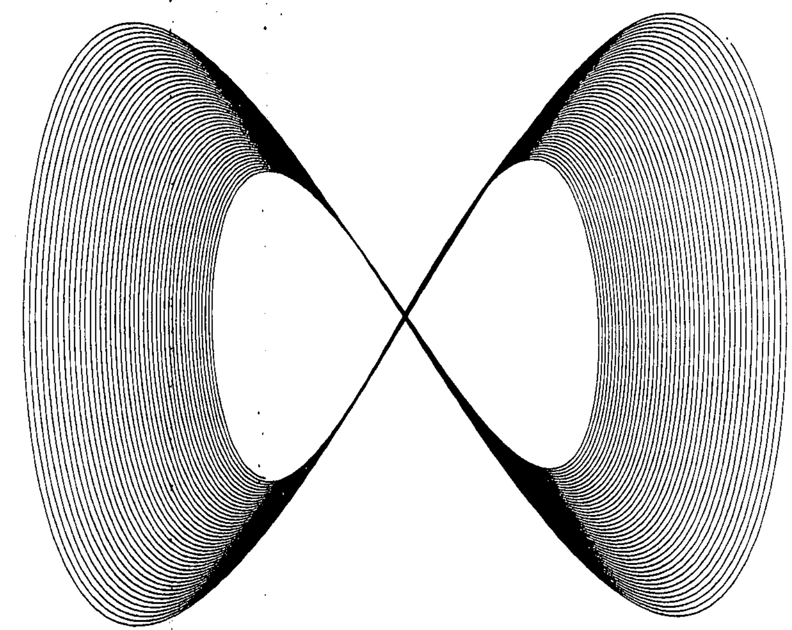
Unfortunately this means that for the octave, and other ratios where one pendulum is going much faster than the other, the theoretical markers have to be ignored, and the right point found by trial and error.

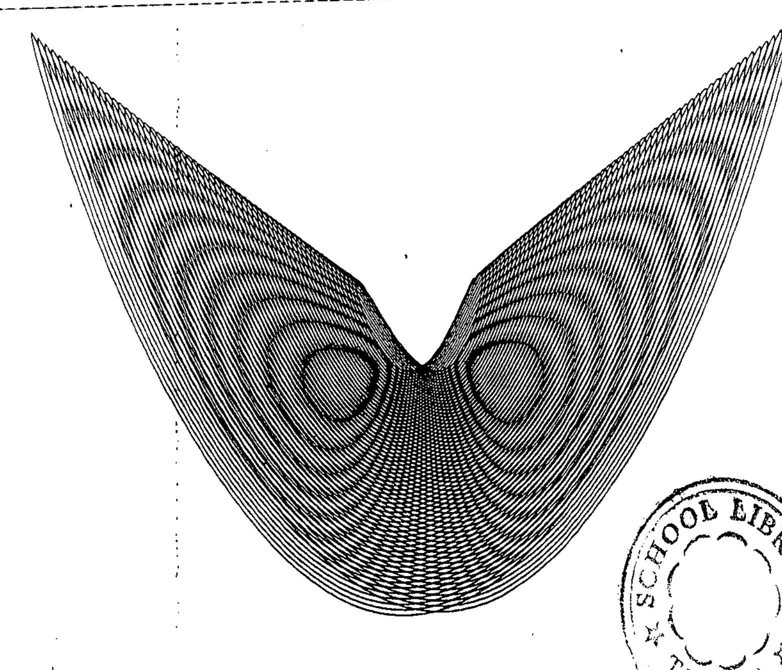
With one pendulum beating twice as fast and at right angles to the other, the octave in open phase takes the form of a figure eight (a coincidence), repeated in diminishing size as the pendulum runs down.

If both pendulums are released at the same time to produce the closed phase, the result is a cup-shaped line that develops into a beautiful winged form with fine cross-hatchings and interference patterns. Small adjustments produce striking variations.

The octave is the first overtone (*see page 8*).







The Rotary Octave—2:1

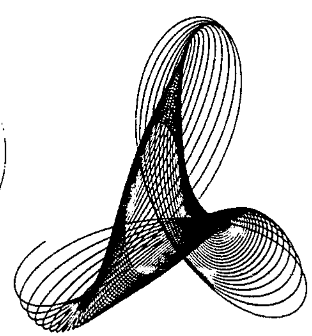
hearts and triangles

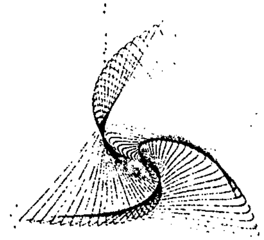
From rotary motion with a 2:1 ratio come some of the most beautiful of all harmonograph drawings: simple, graceful, and often surprising. Remember, all that is happening here is that two circular motions, one almost exactly twice as fast as the other, are being combined.

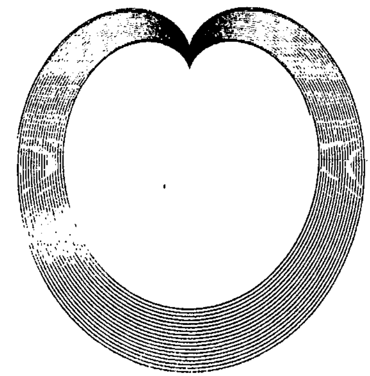
Contrary motion produces a trefoil shape with many fine variations (*opposite, right*). Starting with a smaller size or amplitude in the faster rotation produces a triangle, or pyramid.

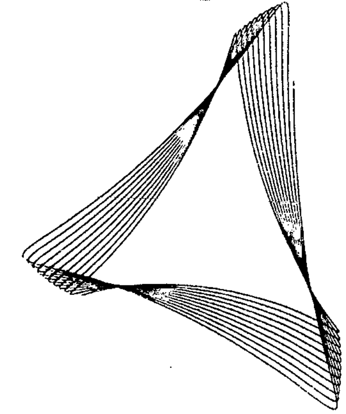
The octave in concurrent motion produces a heart-shaped form with a simple inner loop (*below, left and opposite, left*). Here there is a link with the ancient tradition of the music of the spheres, for this is the shape an observer on Uranus would ascribe to the movement of Neptune, or vice-versa. This is because the planets orbit the Sun concurrently, Uranus in 84 years and Neptune in 165, approximately representing an octave.

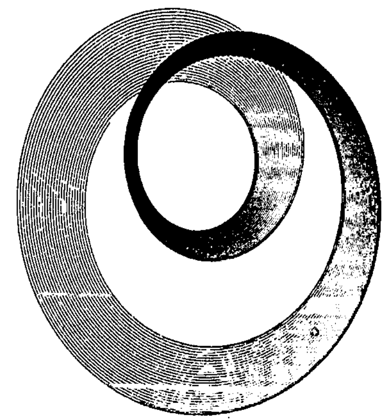
Near misses in the ratios of rotary drawings set the designs spinning (*opposite, bottom*).

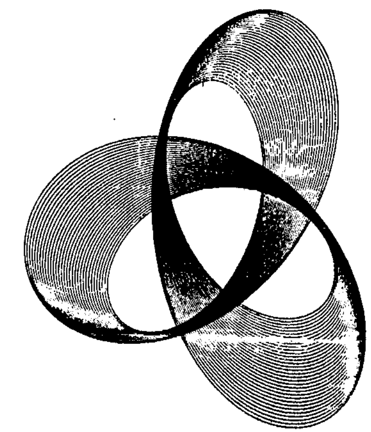


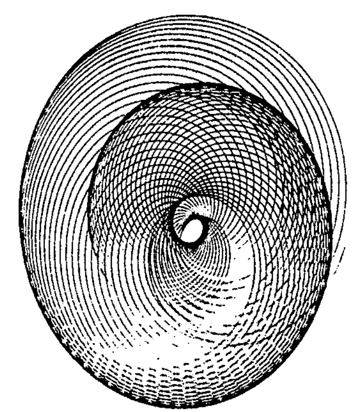


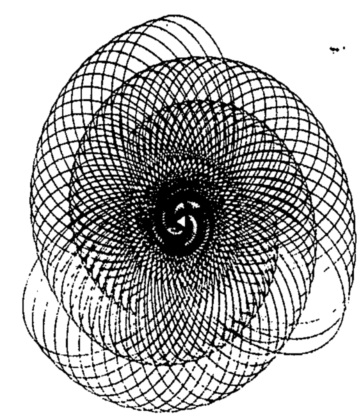












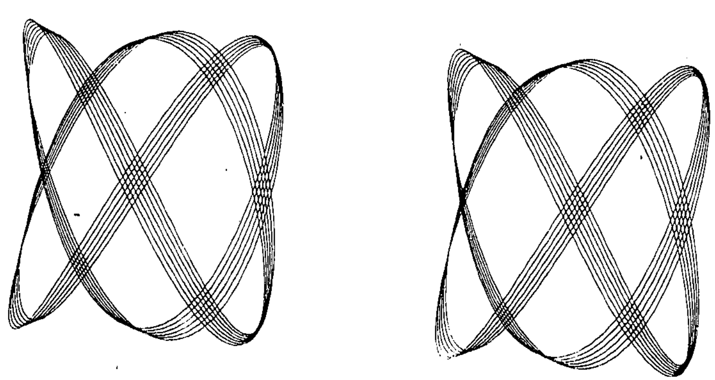
THE LATERAL FIFTH—3:2

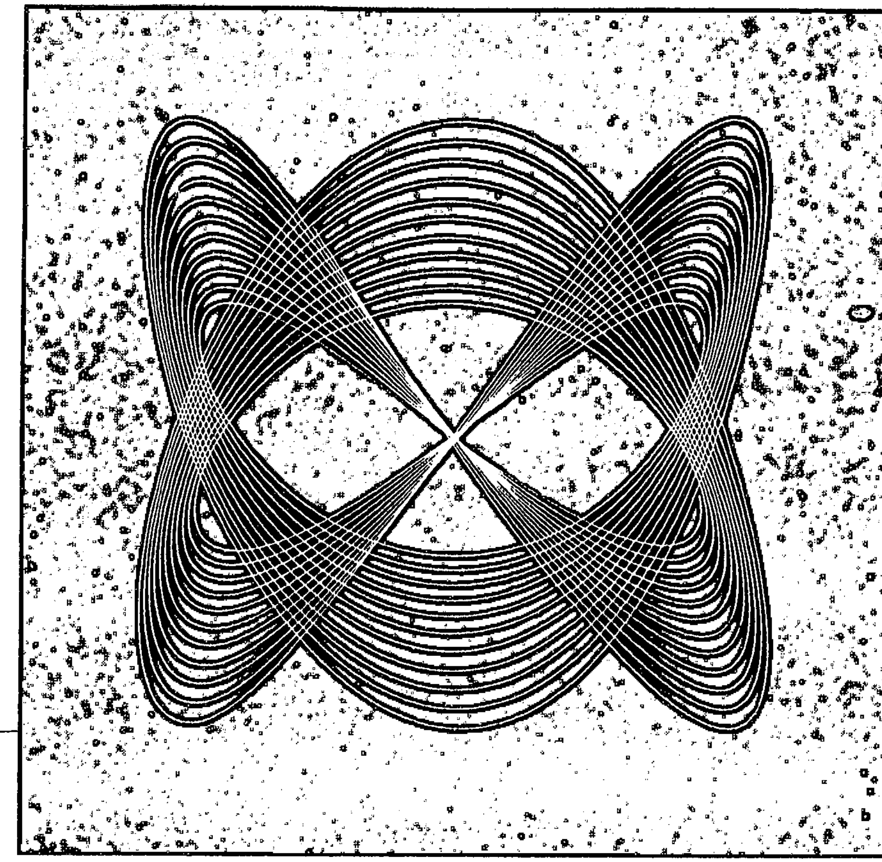
and the second overtone 3:1

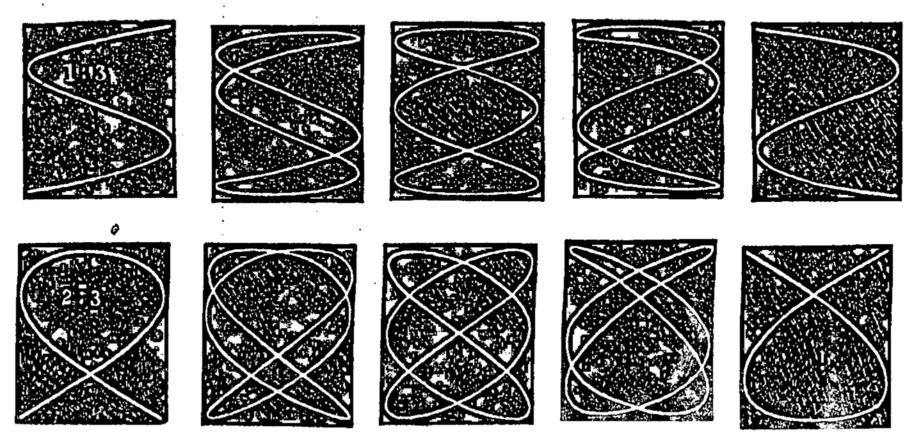
Next to be tried is the harmony of the fifth, intermediate between the simplicity of unison and octave and the more complex harmonies that follow.

It will be seen from the open phase drawing opposite that the fifth has three loops along the horizontal side and two along the vertical. The number of loops on each side gives the ratio, 3:2. Looking back at the octave, there are two loops to one, and with unison there is only one loop, however you look at it. This is the general rule for all lateral harmonograph ratios, and if a harmony appears unexpectedly during experiments, it can usually be identified by counting the loops on two adjacent sides.

The fifth also appears as 3:1, the second overtone, a fifth above the octave (*see open- and closed-phase drawings of 3:1 on page 3*). Drawing ratios outside the octave may require a twin-elliptic harmonograph (*see page 58*). The phase-shifted pair below are stereographic: if you go cross-eyed they will appear three dimensional.







The Rotary Fifth—3:2

encircled hearts and fives

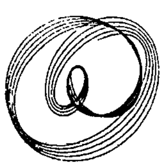
The loudness of musical tones is represented on the harmonograph by *amplitude*, the relative sizes of the two circular motions. In rotary drawings this is much more important than phase, which simply orients the whole design on the page.

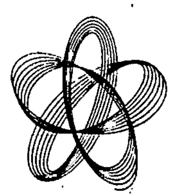
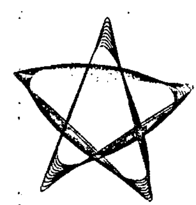
The third drawing below shows a rotary fifth in contrary motion where the higher-frequency, faster-moving pendulum has a much wider swing. In the spiky drawing to its right it is the other way around. At equal amplitude all lines pass through the center *(see table on page 55)*.

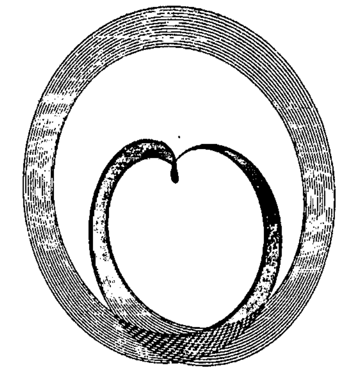
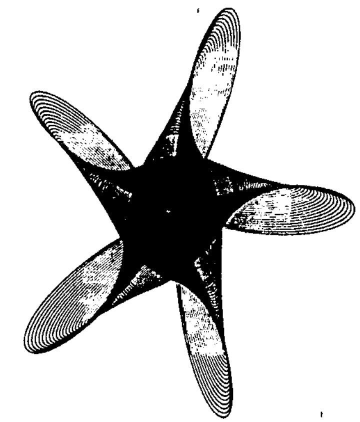
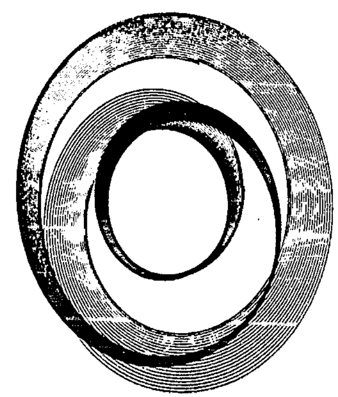
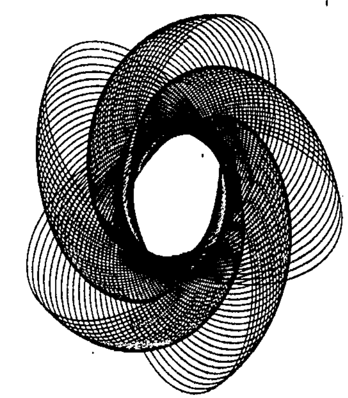
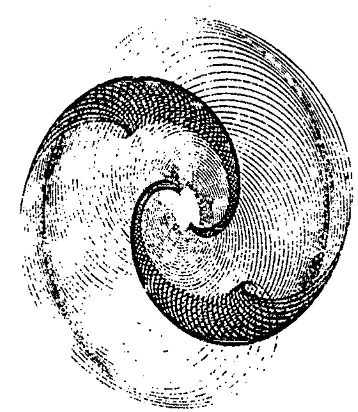
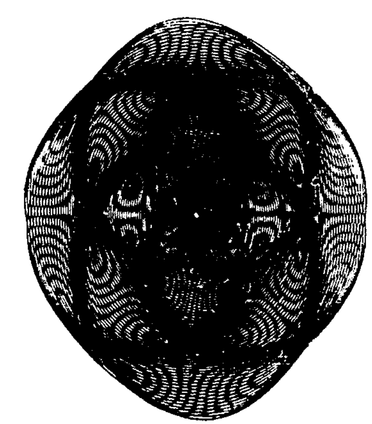
The top four drawings opposite show rotary forms of 3:2, concurrent on the left, and countercurrent on the right. The second row shows the effect of a near miss in the harmony, which makes the patterns spin.

The lower two images opposite are of the second overtone, 3:1, a fifth above the octave (3:1 = 2:1 x 3:2). The concurrent version is on the left, countercurrent on the right.

With concurrent pictures, the number of swirls in the middle is given by the difference between the two numbers of the ratio. So the concurrent patterns for the primary musical intervals 2:1, 3:2, 4:3, 5:4, and 5:6 all have a single heart at their center.



THE FOURTH—4:3

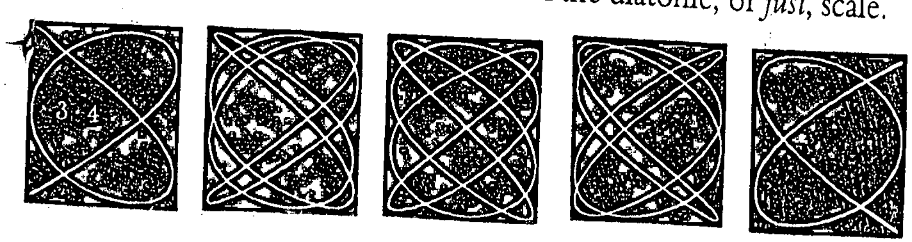
with thirds, sixths, and sevenths

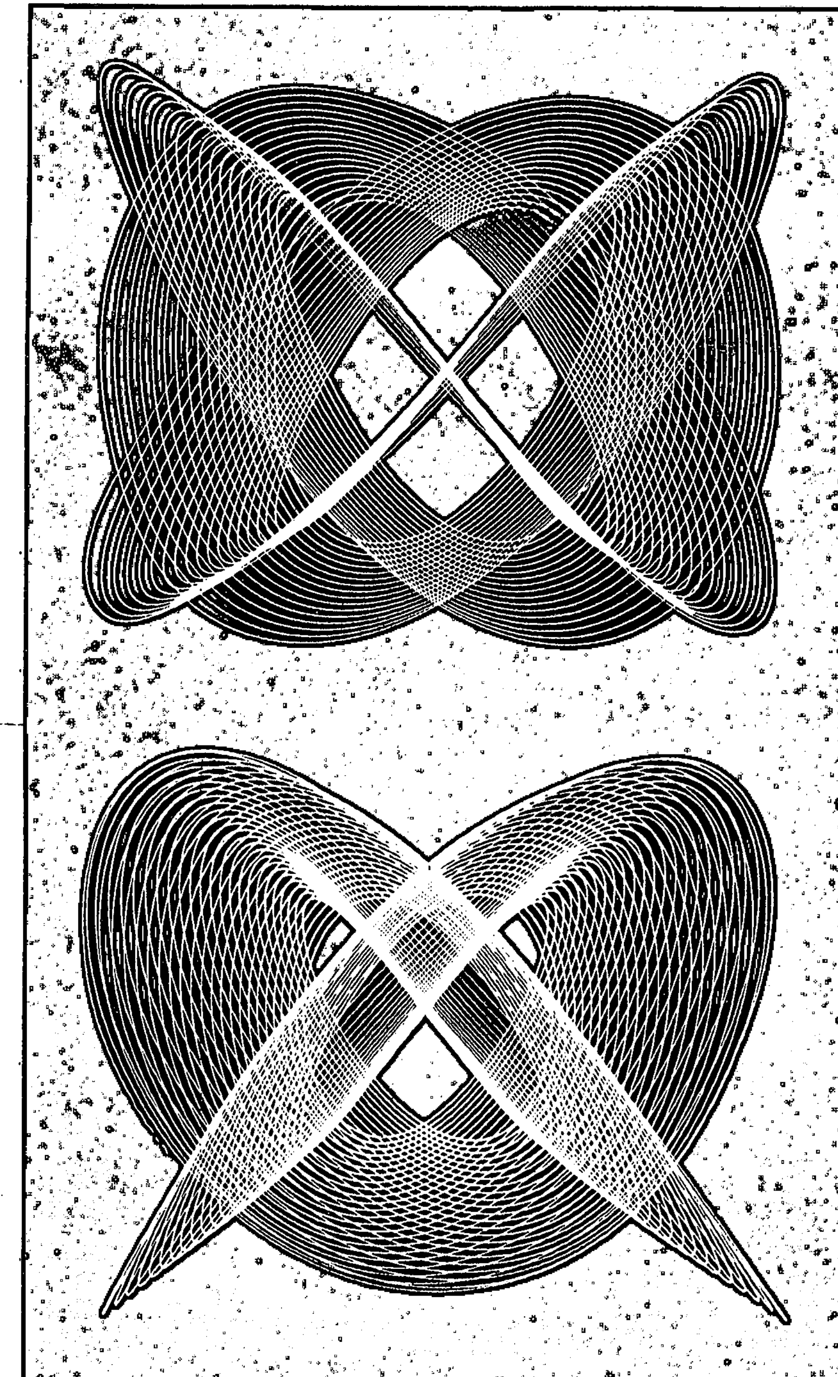
By now it will be evident that each harmony displays its own distinct aesthetic character. Unison is simple and assertive. The octave introduces an emphatic flourish, and the fifth, while still fairly simple, has added elegance.

With the fourth the pattern becomes more complicated, though the design is still recognizable without counting the loops. The upper diagram opposite shows the fourth in open phase, the lower in closed phase. An increasing sophistication becomes apparent, and some of the closed phase and near-miss variants have a strange, exotic quality.

Introducing the perfect thirds of diatonic tuning increases the complexity. The major third (5:4) is found below the fourth, the interval between them, a *diatonic halftone*, working out as 4:3 ÷ 5:4 = 16:15. A fourth and a major third (4:3x5:4) produce the major sixth, 5:3, a minor third (6:5) below the octave and a minor whole tone (10:9) above the fifth. Likewise, a fourth and a minor third (4:3x6:5) create the minor sixth (8:5), a major third (5:4) below the octave and a halftone (16:15) above the fifth.

A fifth and a major third (3:2x5:4) produce the major seventh, 15:8, while a fifth and a minor third (3:2x6:5) give the minor seventh, 9:5. These are the elements of the diatonic, or *just*, scale.





FURTHER HARMONICS

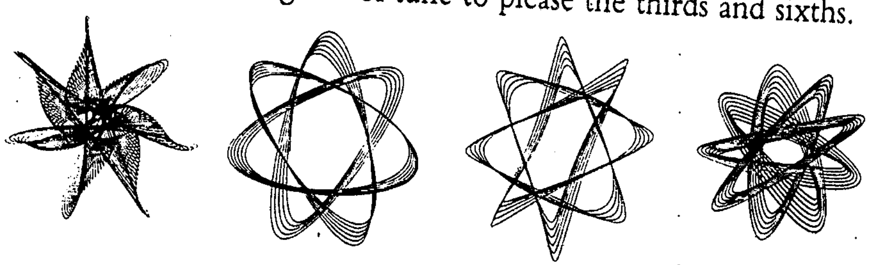
seven-limit and higher-number ratios

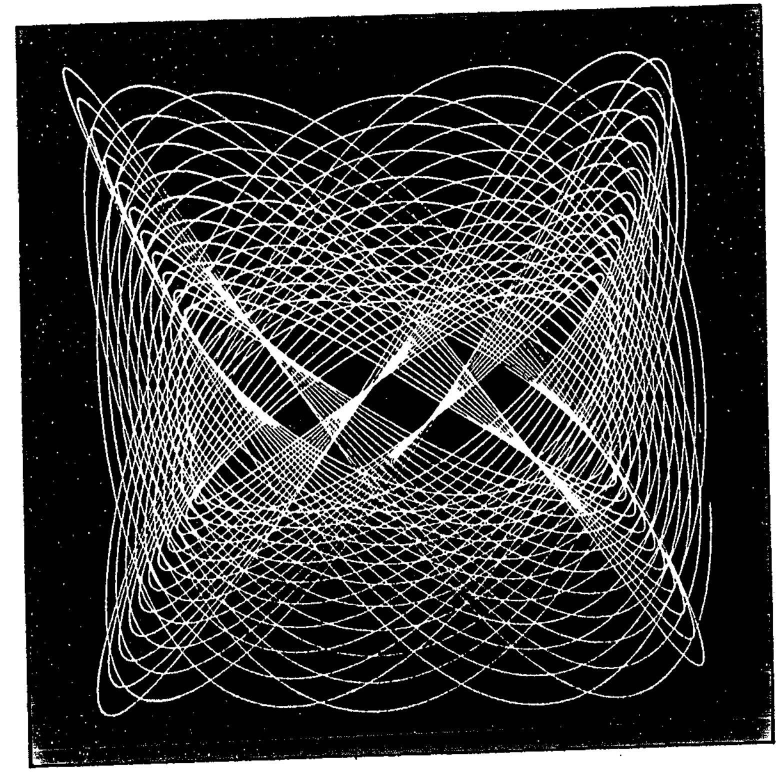
As the numbers in the ratios increase it becomes harder to distinguish the harmonies one from another at a glance: The loops have to be counted, and slight variations produce little of aesthetic value. A typical example, 7:5, is shown opposite top.

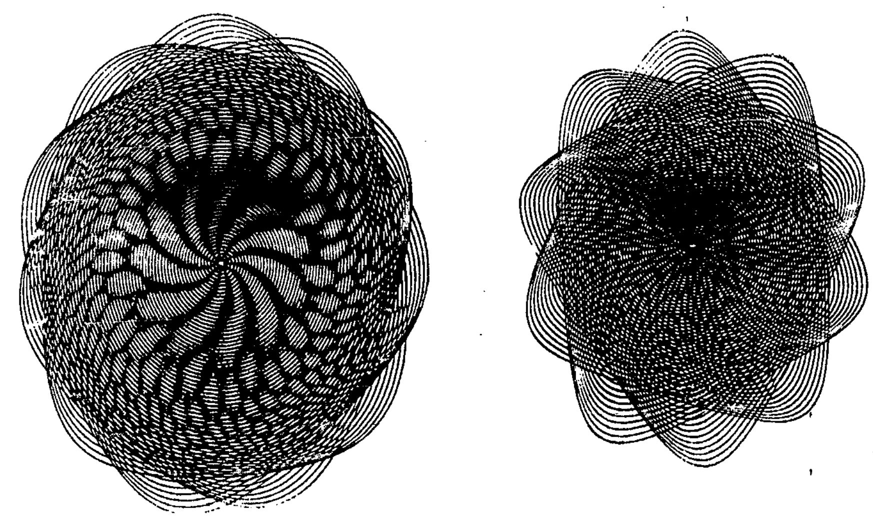
Rotary motion produces a series of increasingly complex drawings, influenced by relative frequency, amplitude, and direction. In contrary motion the total number of loops equals the sum of the two numbers of the ratio. With concurrent motion the nodes turn inward, and their number is equal to the difference between the two numbers of the ratio.

The contrary drawings below show a fourth (4:3), another fourth, a major sixth (5:3) and a major third (5:4). The pictures opposite show unequal amplitude drawings of the perfect eleventh 8:3 (an octave and a fourth) and the ratio 7:3 which is found in seven-limit tuning (not covered in this book).

Two octaves and a major third (4:1 x 5:4) equal 5:1, the fourth overtone, which differs from four fifths $(3:2)^4$ as 80:81, the syntonic comma (*see page 10*). In *mean tone* tuning, popular during the Renaissance, the fifths were flattened very slightly, to $5^{1/4}$ or 1.4953, falling out of tune to please the thirds and sixths.







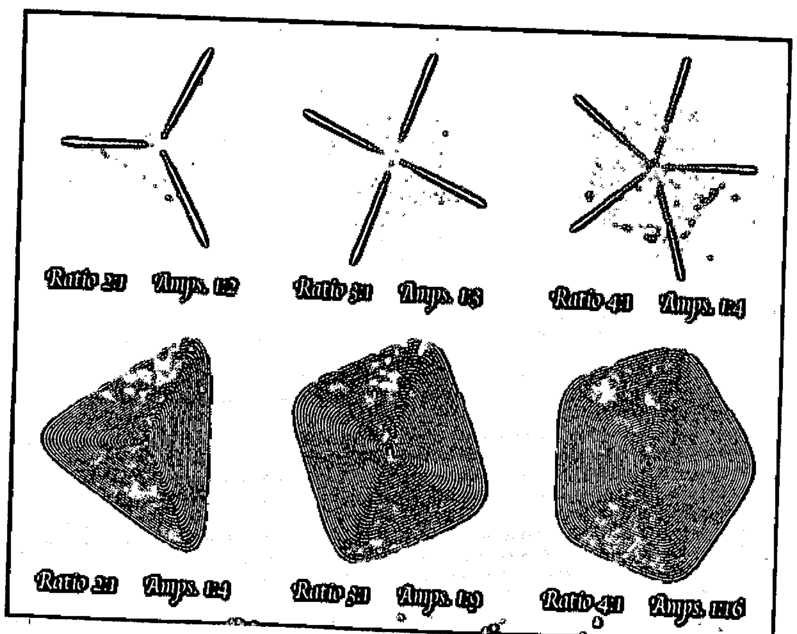
Amplitude

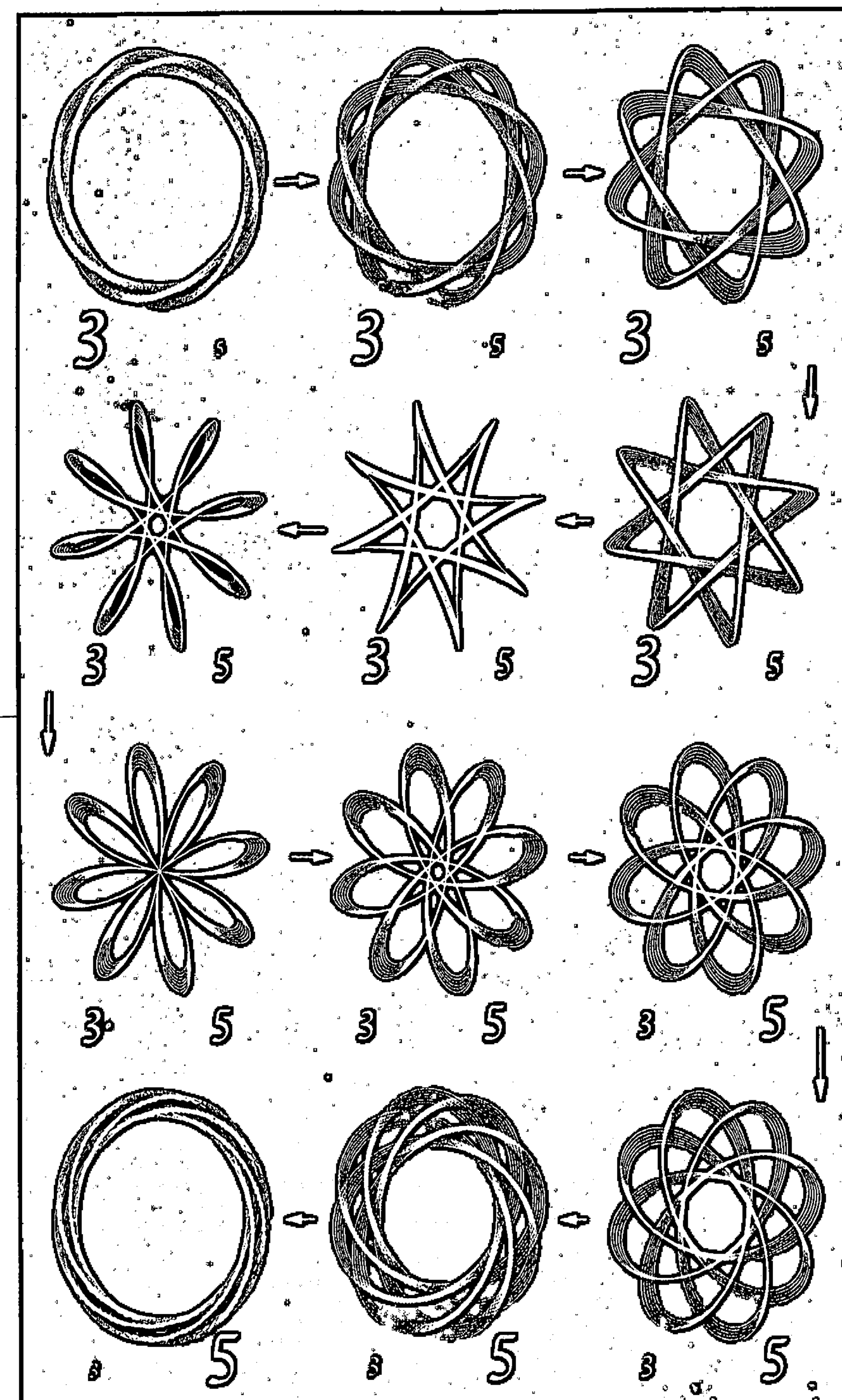
circles, polygons, flowers, and another circle

Much variation can be obtained from a rotary ratio by having unequal sizes in the two circular motions. Opposite we see two frequencies related by a major sixth (5:3). A lower-frequency note begins to be influenced by, combines with, and is then more or less replaced by a higher-frequency one. When the two notes are at equal volume the lines all pass through the center (*see pages 56–57*). Notice that the sequence is not symmetrical.

Below we see the first three overtones. For the spikiest shapes simply invert the amplitudes. For polygons, square them first.

If you have ever played with a spirograph, the harmony is determined by the cogging ratio, and it is the amplitude that is adjusted when you change penholes on the wheel.





TUNING TROUBLES

the Pythagorean comma

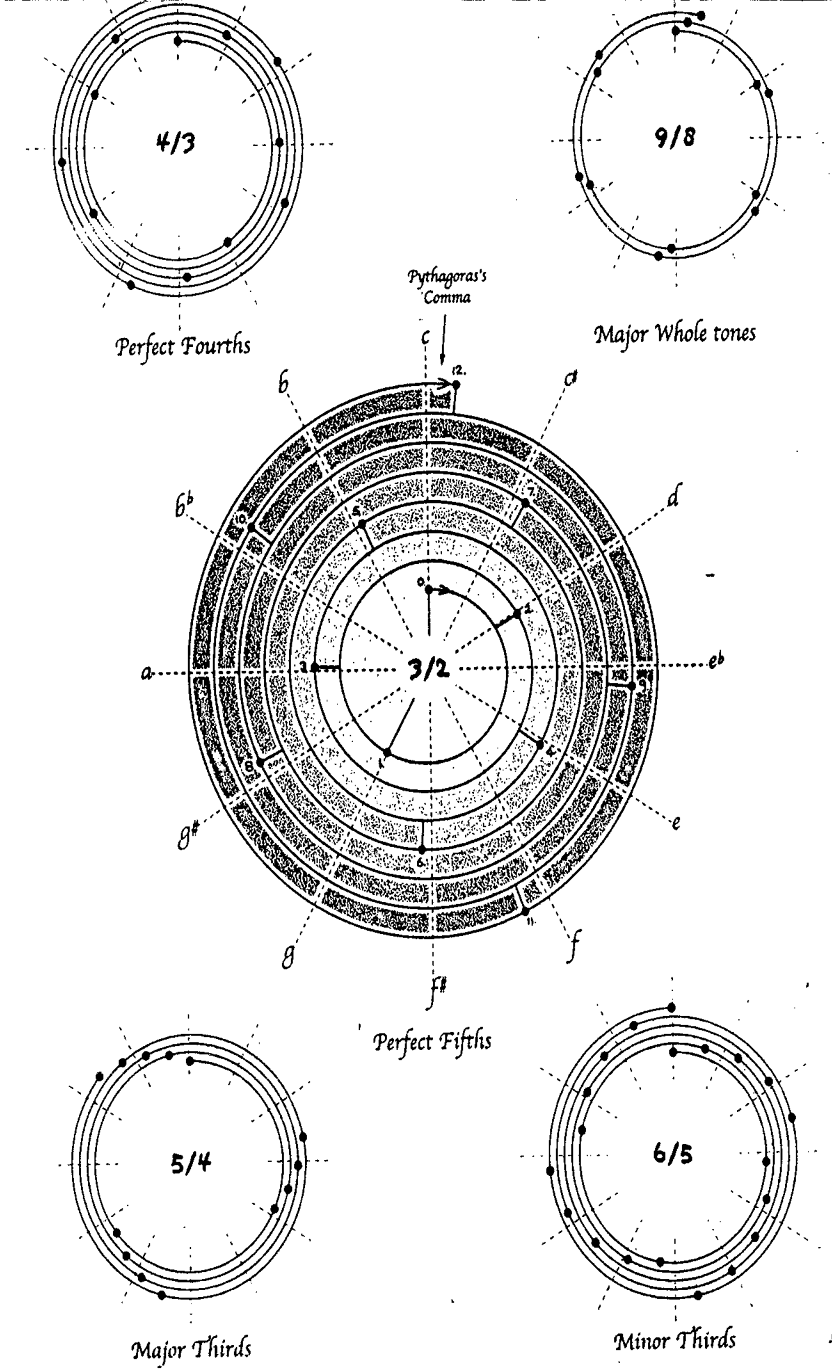
Leaving the harmonograph drawings and returning to the principles of music, you may have noted that musical intervals do not always agree with one another. A famous example of this is the relationship between the octave and the perfect fifth (3:2).

In the central picture opposite, a note is sounded in the middle at 0, and moved up by perfect fifths (*numbered opposite, each turn of the spiral representing an octave*). After twelve fifths it has gone up seven octaves, but the picture shows that it has overshot the final octave slightly, and gone sharp. This is because $(3/2)^{12} \approx 129.75$, whereas $(2)^7 = 128$. The difference is known as the Pythagorean comma, 1.013643—approximately 74:73.

If you kept on spiraling you would eventually discover, as the Chinese did long ago, that 53 perfect fifths (or *Lü*) almost exactly equal 31 octaves. The first five fifths produce the pattern of the black notes on a piano, the *pentatonic* scale (*see page 50*).

The smaller pictures opposite show repeated progressions of the major third (5:4), the minor third (6:5), the fourth (4:3), and the whole tone (9:8) all compared to an invariant octave.

It's strange. With all this harmonious interplay of numbers you would have expected the whole system to be a precisely coherent whole. It isn't. There are echoes here from the scientific view of a world formed by broken symmetry, subject to quantum uncertainty, and (so far) defying a precise comprehensive theory of everything. Is this why the near miss is so often more beautiful than perfection?



Equal Temperament

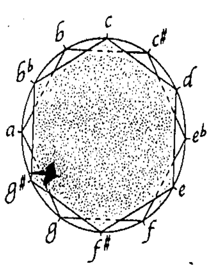
changing keys made easy

Although early tunings enabled many pure harmonic ratios to be played, it was often hard to move into other keys; all one could do easily was change *mode* (*see page 52*). Musicians often had to retune their instruments, or use extra notes reserved for specific scales (classical Indian tuning uses twenty-two notes).

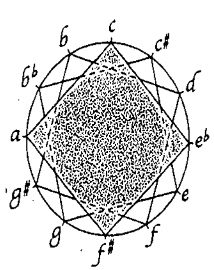
In the sixteenth century a new tuning was developed that revolutionized Western music and that predominates today. The octave is divided into twelve equal intervals, each *chromatic semitone* being 1.05946 times its neighbor ($2^{1/12}$, roughly 18:17).

Twelve equally spaced notes are arranged in a circle below. Six (flat) whole tones now make an octave, as do four (very flat) minor thirds, or three (sharp) major thirds. The Pythagorean comma vanishes, as do all perfect intervals except the octave. It's a clever fudge, slightly out of tune and we hear it every day.

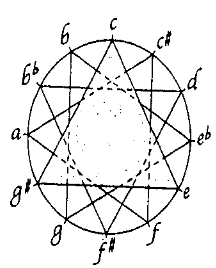
Triads are chords of three notes. Opposite top we see major and minor triads involving the note C, in the key of C. Use the master grid (*opposite below*) to navigate the even-tempered sea, and place any triad in three distinct keys (*after Malcolm Stewart*).



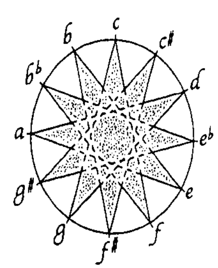
whole tones and sevenths



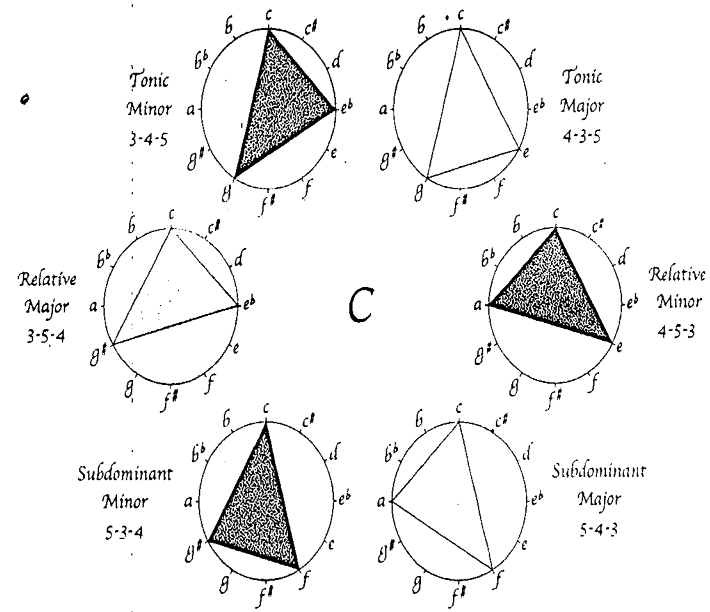
minor thirds and major sixths

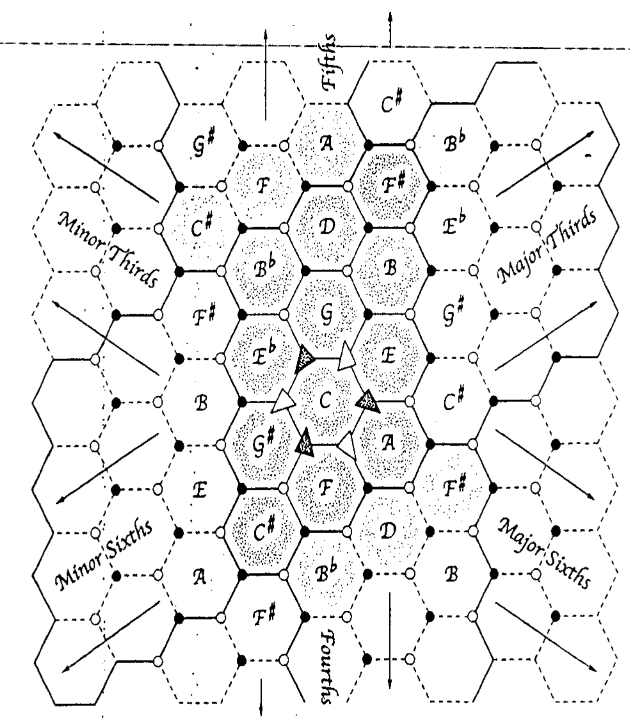


major thirds and minor sixths



fifths and fourths





THE KALEIDOPHONE

squiggles from a vibrating rod

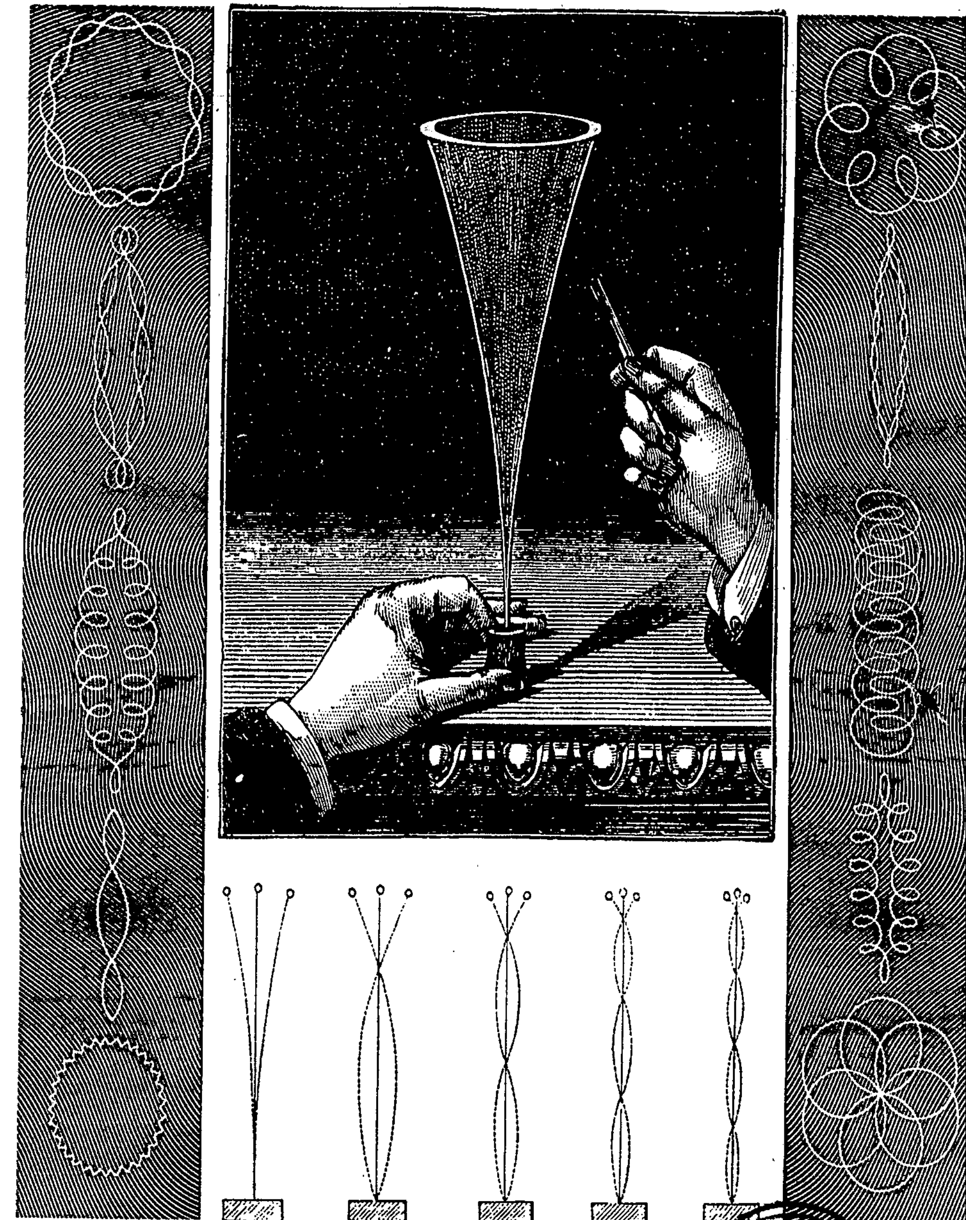
Despite the invention of equal temperament, scientists continued to investigate pure ratio harmonics. An interesting nineteenth-century precursor to the harmonograph was the kaleidophone, invented by Sir Charles Wheatstone in 1827. Like the harmonograph, it displayed images of harmonics.

The simplest version of the device consists of a steel rod with one end firmly fixed into a heavy brass stand and the other fixed to a small silver glass bead, so that when illuminated by a spotlight a bright spot of light is thrown up on a screen placed in front of it. Depending on how the kaleidophone is first struck, and then subsequently stroked with a violin bow, a surprising number of patterns can be produced (*a few are shown opposite*).

The kaleidophone does not behave like a string, as it is only fixed at one end. Like wind instruments, which are normally open at one end, the mathematics of its harmonics and overtones are more complicated than the monochord or the harmonograph (*the lower images opposite show some early overtones*).

Other versions of the kaleidophone used steel rods with square or oval cross sections to give further patterns. Wheatstone used to refer to his invention as a "philosophical toy," and indeed, as we look at these patterns, it is easy to feel wonder at their simple beauty.

To make your own kaleidophone, try fixing a knitting needle in a vise and attaching a silver bead to the free end. Use or make a light source that projects a bright point of light.



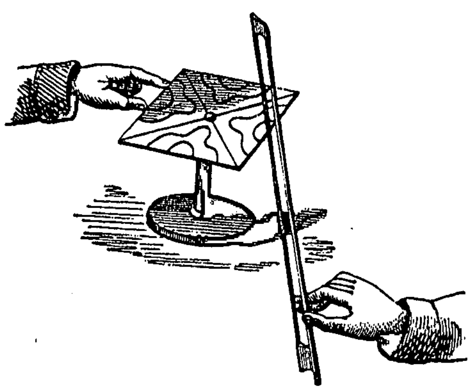
Chladni Patterns

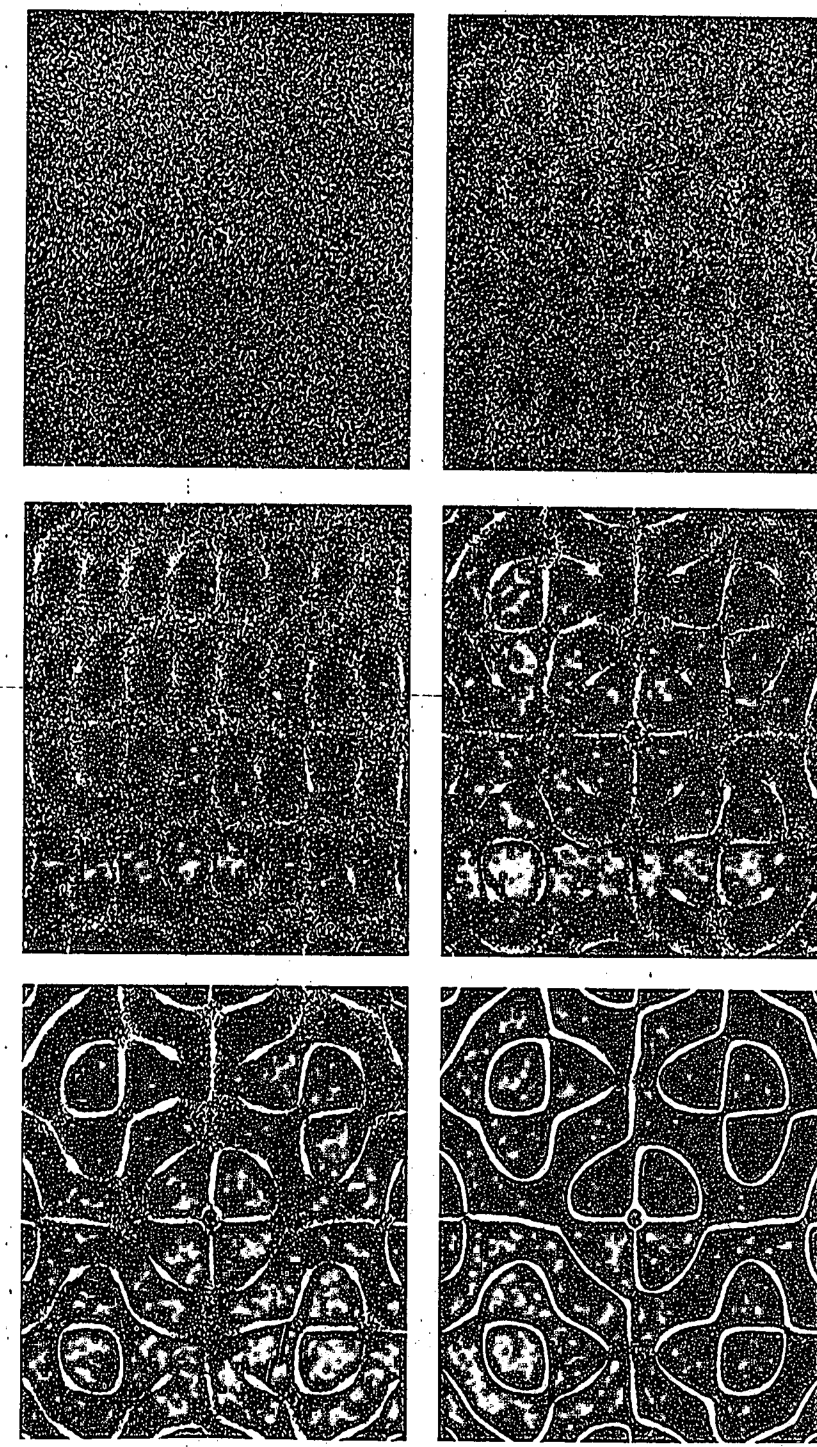
vibrating surfaces

So far we have only considered vibrating strings and other simple systems, but surfaces also can be made to vibrate, and they too can display harmonic or resonant patterns.

In 1787 Ernst Chladni found that if he scattered fine sand onto a square plate, and bowed or otherwise vibrated it, then certain notes, generally harmonics of each other, each gave rise to different patterns in the sand on the plate. Like the harmonograph, other disharmonic tones produced a chaotic mess. Sometimes he found that further patterns could be created by touching the side of the plate at harmonic divisions of its length (*shown below*). This created a stationary node (*see page 8*). Later work revealed that circular plates gave circular patterns, triangular plates triangular patterns and so on.

The six pictures opposite are from Hans Jenny's book *Cymatics*, one of the seminal texts on this subject. The vibration picture appears gradually, the sand finding its way to the stationary parts of the plate as the volume increases.





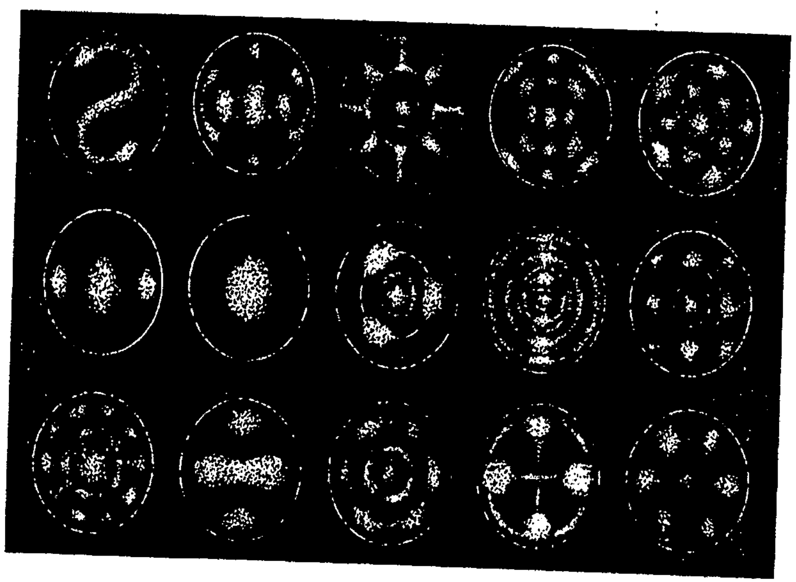
Resonance Pictures

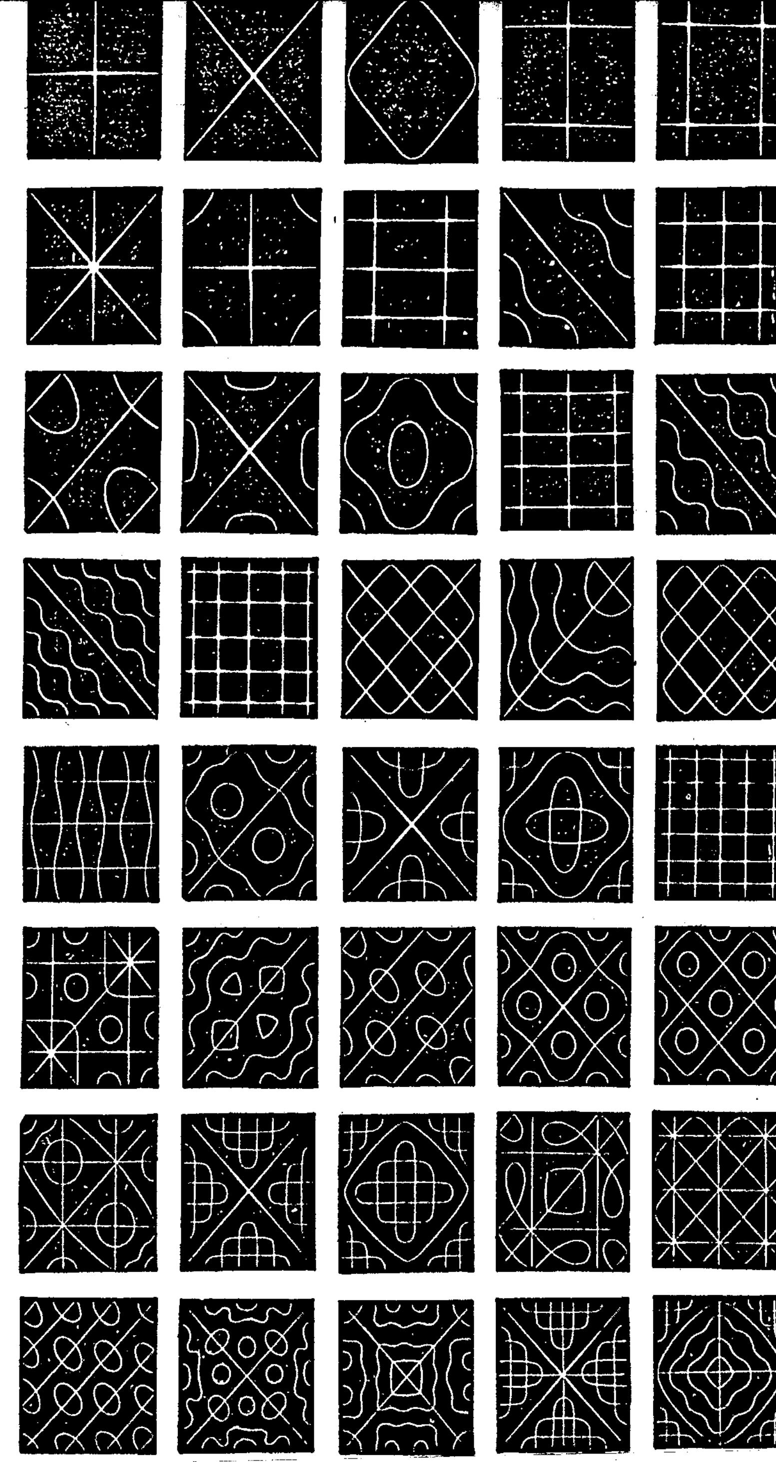
and how to sing a daisy

A more complete set of Chladni figures is shown opposite, all two- or fourfold because they were produced on a square plate.

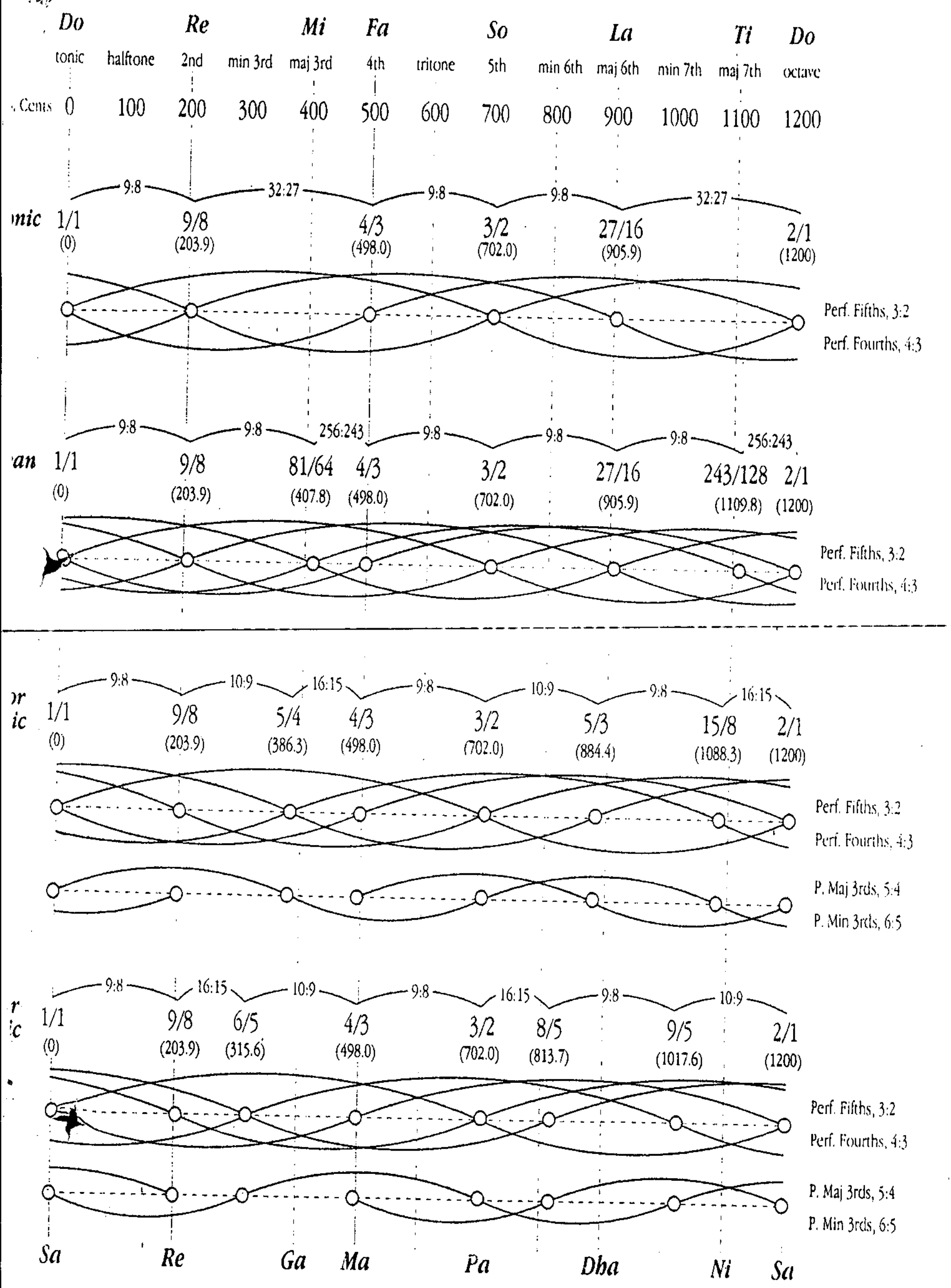
Below, however, we see some circular patterns. They were photographed in the 1880s by Margaret Watts Hughes, a singer, on an ingenious device called an eidophone, which consisted of a hollow base with a membrane stretched across it and a tube attached to its base with a mouthpiece at the other end. As Mrs. Hughes sang diatonic scales down the tube, fine lycopodium powder scattered on the taut membrane suddenly came to life, bouncing away from some places and staying still at others, producing shapes that she likened to various flowers.

Yet again, we see recognizable forms and shapes appearing from simple resonance and harmony.

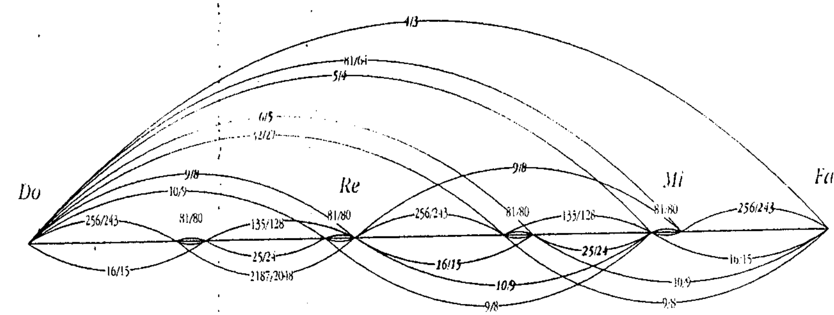




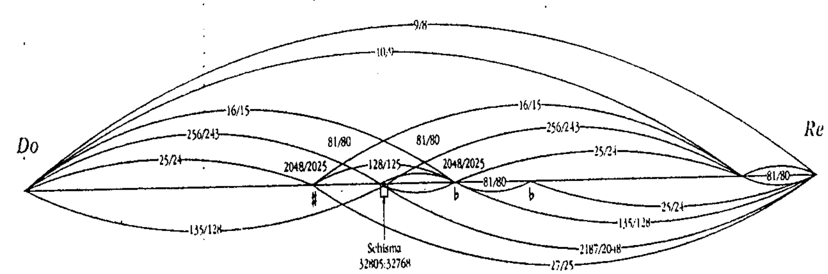
Appendix A: Tunings and Intervals



Selected Musical Intervals

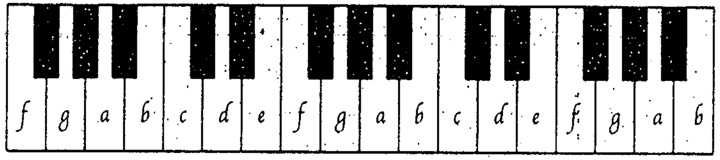


Interval	*Cents*	*Name*	6:5	315.6	*Perfect Minor Third*
1:1	0	*Unison*	5:4	386.3	*Perfect Major Third*
32805:32768	2.0	*Schisma*	81:64	407.8	*Pythagorean Major Third*
2048:2025	19.6	*Diaschisma*	4:3	498.0	*Perfect Fourth*
81:80	21.5	*Syntonic Comma*	7:5	582.5	*Septimal Tritone*
531441:524288	23.5	*Pythagorean Comma*	45:32	590.2	*Diatonic Tritone*
128:125	41.1	*Diesis*	729:512	611.7	*Pythagorean Tritone*
25:24	70.7	*Minor Diatonic Halftone*	3:2	702.0	*Perfect Fifth*
256:243	90.2	*Leimma, Pythag. Halftone*	128:81	792.2	*Pythagorean Minor Sixth*
135:128	92.2	*Major Chroma*	8:5	813.7	*Diatonic Minor Sixth*
16:15	111.7	*Major Diatonic Halftone*	5:3	884.4	*Perfect Major Sixth*
2187:2048	113.7	*Apotome*	27:16	905.9	*Pythagorean Major Sixth*
27:25	133.2	*Large Leimma*	7:4	968.8	*Harmonic Seventh*
10:9	182.4	*Minor Whole tone*	16:9	996.1	*Pythagorean Minor Seventh*
9:8	203.9	*Major Whole tone*	9:5	1017.6	*Diatonic Minor Seventh*
8:7	231.2	*Septimal Whole tone*	15:8	1088.3	*Diatonic Major Seventh*
7:6	266.9	*Septimal Minor Third*	243:128	1109.8	*Pythagorean Major Seventh*
32:27	294.1	*Pythag. Minor Third*	2:1	1200	*Octave*



Like the off-center division of the octave into fifths and fourths, sharps are not in fact flats, giving rise to five more notes, making seventeen in all (found in Middle-Eastern tunings). More completely, we may think of the seven notes of the scale as moving across twelve "regions" of the octave, falling into the twenty-two positions of Indian tuning.

Appendix B: Modes and Equations



The Seven Modes of Antiquity

Modern Names	Steps	Solfège	Notes	Degrees	Ancient Greek Names
Ionian *Major*	1 1 ½ 1 1 1 ½	do re mi fa so la ti do	c d e f g a b c	1 2 3 4 5 6 7 8	Lydian
Dorian	1 ½ 1 1 1 ½ 1	re mi fa so la ti do re	d e f g a b c d	1 2 3♭ 4 5 6 7♭ 8	Phrygian
Phrygian	½ 1 1 1 ½ 1 1	mi fa so la ti do re mi	e f g a b c d e	1 2♭ 3♭ 4 5 6♭ 7♭ 8	Dorian
Lydian	1 1 1 ½ 1 1 ½	fa so la ti do re mi fa	f g a b c d e f	1 2 3 4♯ 5 6 7 8	Syntolydian
Myxolydian	1 1 ½ 1 1 ½ 1	so la ti do re mi fa so	g a b c d e f g	1 2 3 4 5 6 7♭ 8	Ionian
Aeolian *Natural Minor*	1 ½ 1 1 ½ 1 1	la ti do re mi fa so la	a b c d e f g a	1 2 3♭ 4 5 6♭ 7♭ 8	Aeolian
Locrian	½ 1 1 ½ 1 1 1	ti do re mi fa so la ti	b c d e f g a b	1 2♭ 3♭ 4 5♭ 6♭ 7♭ 2	Myxolydian

The white notes on a piano give the seven notes of the seven modes of ancient Greece. Medieval transcription errors have left us with modern names that don't fit the ancient ones. Each mode, or scale, has its own pattern of whole tones and halftones, only two surviving as our major and natural minor scales.

Other scales include modal pentatonics that forbid semitones, the harmonic minor with its minor 3rd and 6th, 1 2 3♭ 4 5 6♭ 7 8, and many others.

The ratios and intervals in this book concern frequencies, normally expressed as cycles per second, or Hertz. Classical tuning sets C at 256 Hz. Modern tuning is higher, fixing A at 440 Hz. The period T of a wave is the reciprocal of its frequency f: $T = 1/f$.

The speed of sound in dry air is roughly $331.4 + 0.6T_c$ m/s, where T_c is the temperature in degrees celsius. Its value at room temperature, 20°C, is 343.4 m/s.

Gravitational acceleration on Earth, g, is 9.807 m/s².

Frequency of a pendulum: $\frac{1}{2\pi}\sqrt{\frac{\text{gravitational acceleration}}{\text{pendulum length}}}$

Fundamental frequency of a tensioned string: $\frac{1}{2 \times \text{string length}}\sqrt{\frac{\text{string tension}}{\text{string mass} \div \text{string length}}}$

Resonant frequency of a cavity with an opening: $\frac{\text{speed of sound}}{2\pi}\sqrt{\frac{\text{area of opening}}{\text{volume of cavity} \times \text{length of opening}}}$

Fundamental frequency of an open pipe or cylinder: $\frac{\text{speed of sound}}{2 \times \text{length of cylinder}}$

The beat frequency between f_1 and f_2 is the difference between them, $f_b = f_2 - f_1$.

The ratio a:b converts to cents (where a > b): (log(a)-log(b)) x (1200 ÷ log2).
To convert cents into degrees multiply by 0.3.

Clapping in front of a rise of steps produces a series of echoes with a perceived frequency equal to $v/2d$, where v is the speed of sound, and d is the depth of each step. Clapping in a small corridor width w produces a frequency v/w.

The *arithmetic* and *harmonic means* are central to Pythagorean number theory. The arithmetic mean of two frequencies separated by an octave produces the fifth between them (3:2), the harmonic mean producing the fourth (4:3).

$$6 : 8 :: 9 : 12$$

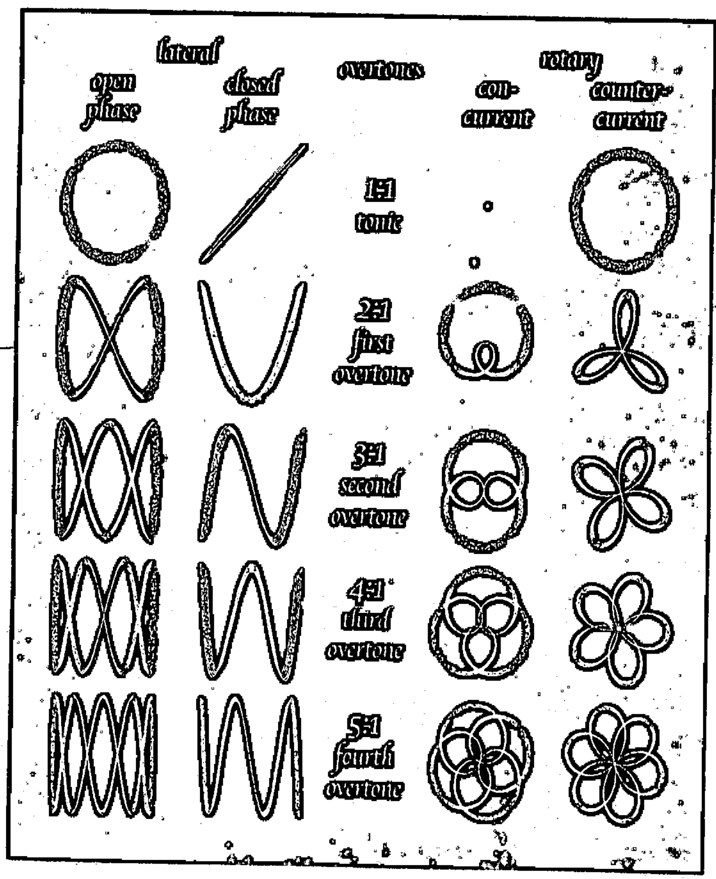
$$A : \frac{A+B}{2} :: \frac{2AB}{A+B} : B$$

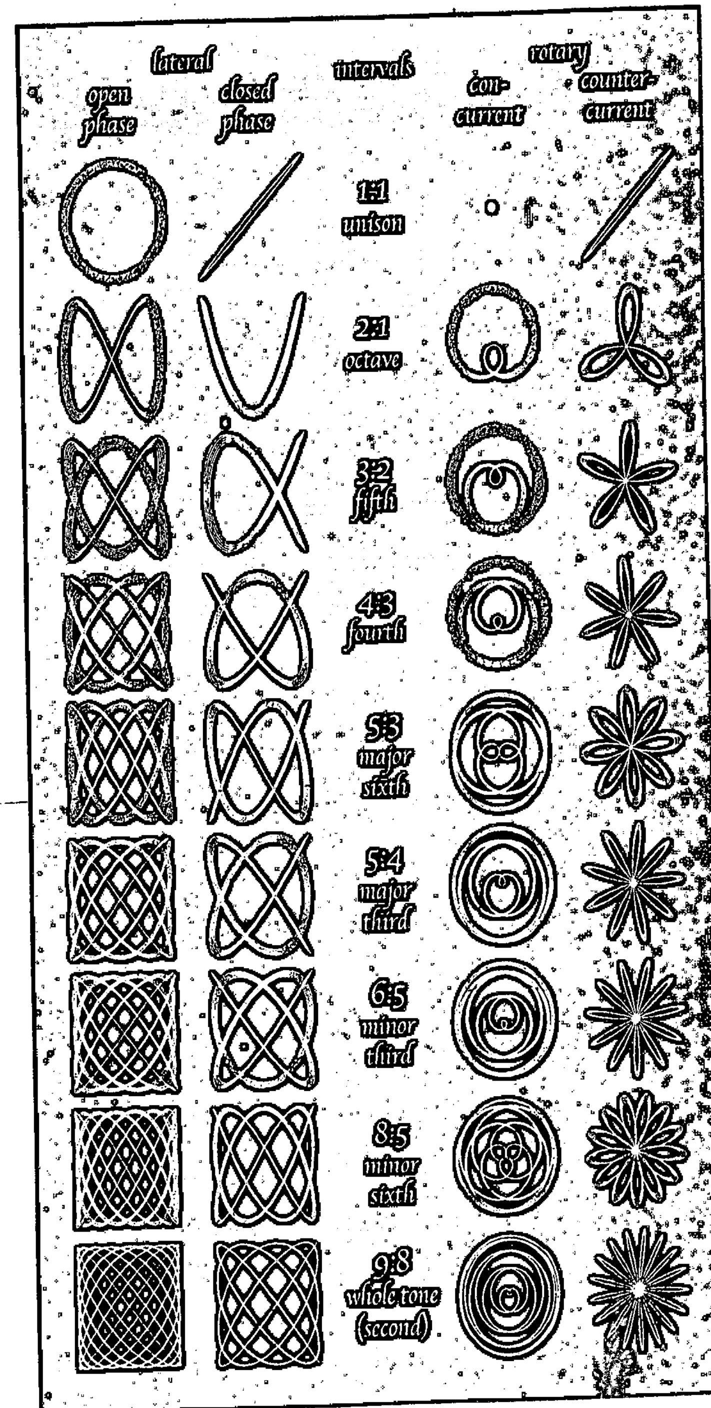
A : Arithmetic Mean :: Harmonic Mean : B

Appendix C: Tables of Patterns

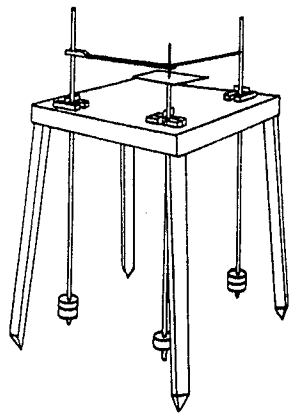
Overtone and simple ratio harmonics are shown below and opposite, arranged in order of increasing dissonance down the page. Open phase drawings display their ratio as the number of loops counted across and down. To find the ratio of a rotary drawing, draw both forms, concurrent (both circles in the same direction) and contrary (in opposite directions). Count the loops in each, add the two numbers together and divide the total by two. This gives the larger ratio number. Subtract this from the contrary total to give the lower ratio number. Rotary figures for the ratio *a*:*b* will have *b*−*a* loops when both circles are concurrent, and *a*+*b* loops when they are contrary.

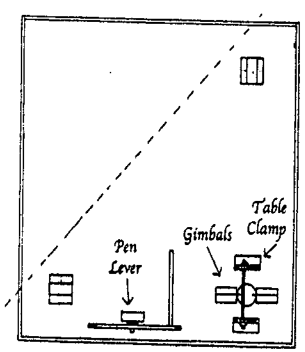
The designs shown here were all made with equal amplitudes.

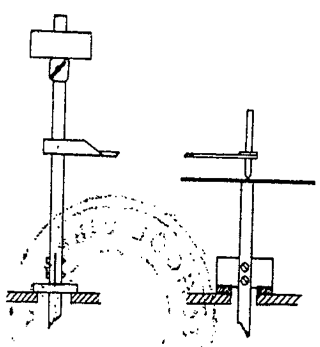




APPENDIX D. BUILDING A HARMONOGRAPH







Anyone seriously interested in making a harmonograph should consider going straight for the three-pendulum model.

The table must be highly rigid and firm on the floor, otherwise the movements of the weights will be distorted. I suggest it should be about 36 inches (90 cm) above the floor, with a tabletop 24 x 12 inches (60 x 30 cm) for two pendulums, 24 x 24 inches (60 x 60 cm) for three, and about ¾ inch (2 cm) thick with an apron all round, about 3¼ inches (8 cm) deep.

The legs should be about 2½ inches (6 cm) square, splayed outward and pointed at the bottom. One way of achieving the splay is to fix wood or metal brackets in the corners under the table on each side of the diagonals and bolt the legs between them. After adjusting the legs to give the correct splay they can then be fixed in position with screws through the apron.

To save space, cut the tabletop as illustrated by the dotted line. Three legs are not quite as stable, but work fairly well.

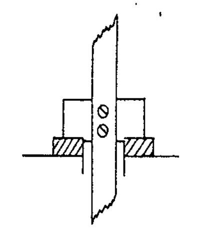
The platform carrying the paper should be light and rigid, and fixed to the pendulum with a countersunk screw. Make the platform about 8⅔ x 6 inches (22 x 15 cm) to hold half an 8½ x 11-inch sheet secured by a rubber band or small clip.

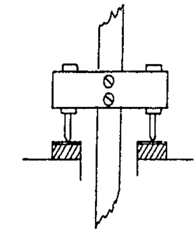
All sizes suggested are maximum, but a scaled-down version will still work if it is carefully made.

If you are tempted to make a harmonograph, start with the weights, for the instrument will only be satisfactory if these are really heavy and yet easy to adjust. It is a good idea to make about ten, around 4 pounds (2 kilos) each, so the loadings can be varied. They should be about 3¼ inches (8 cm) in diameter, with a central hole, or with a slot for easier handling. Either cast them yourself from lead or ready-mixed cement or have them made by a metal shop or plumber.

The shafts should be made from wood dowel, about ½ inch (1.5 cm) in diameter (metal rods are

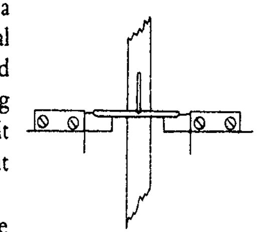
liable to bend, distorting the drawings), marked off in inches.

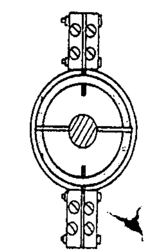




Clamps can be obtained from suppliers of laboratory equipment. For some of the drawings top weights are needed, held in place by clamps. Clamps can also be added to pendulum tops for fine tuning, with one or more metal washers added.

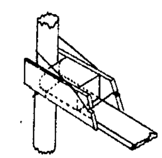
The simpler kind of bearing consists of brass strips bolted into a slot in the pendulum and filed to sharp edges to rest in grooves on each side.

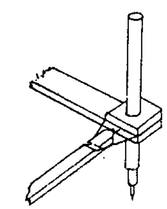




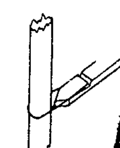
In a bearing involving less friction the pendulum is encased at the fulcrum in a horizontal block of hardwood with vertical bolts on each side filed to sharp points and resting in grooves in metal plates. If drilling the large hole in the block is too difficult, it can be made in two halves, each hollowed out to take the shaft and bolted together.

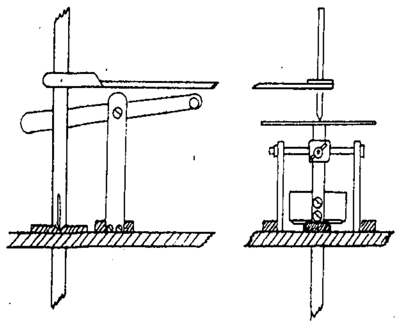
Rotary motion needs gimbals. Here the grooves for the pendulum are filed in the upper side of a ring (e.g. a key ring) while the under side has grooves at right angles to the upper ones. The lower grooves fit on two projecting sharp edges (brass strips), each enclosed between two pieces of wood fixed to the table. With the alternative bearing a large flat washer should be used with depressions to take the sharp points.





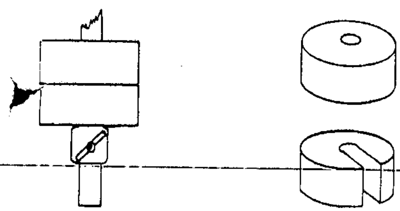
Pen arms should be as light as possible to minimize restraint. They are easily made from balsa wood strips (sold at model-making shops), using glue and Scotch tape. For two pendulums the arm can be fastened to the shaft with pinched-off needles, and the pen jammed into a hole at the other end. For three pendulums the side pieces on the arm should enclose its shaft firmly but not too tightly and be held gently with a thin rubber band. One of the arms holds the pen, while the other is held by protruding needles pushed in backward and secured (gently) at both ends by the rubber bands.





An additional fitting is needed to lock a rotary pendulum so that the instrument can be used with just the two single-axis pendulums. This can be done by mounting two brackets on the table near the rotary pendulum with holes to take a long horizontal bolt (slightly to one side) to which the shaft can be clamped.

Pens should be fine, light, and free-flowing. Most stationers and shops selling draftsmen's and artists' materials offer a variety (avoid ball point or thick fiber pens). For best results use shiny art paper and ordinary copier paper for preliminary experiments.



If the pen is left on the paper to the end there is usually an unsightly blob. To avoid this, mount a short pillar on the table with an adjustable lever carrying a piece of thin dowel placed under the pen arm. By raising the dowel gently the pen is lifted off the paper without jogging it. This device should also be used before the pen is lowered to the paper. By watching the pen you can see what pattern is being made, and nudge it one way or the other by pressure on the pendulums.

For ratios outside the octave, such as 4:1, you may need to try another harmonograph such as Goold's twin-elliptic pendulum (left).